

Fuzzy Rules for Data Set Classification: A Hybrid Approach Using Rough Set and Grid Partitioning

Adeola Azy Daniachew ¹, Avereey Barack Clevon ², Abimelech Keita Avram ³, and Dodavah Tesseman Chislon ⁴

^{1,2}Faculty of Science, University of Cape Town, South Africa

^{3,4}Faculty of Mathematics and Computer Science, Weizmann Institute of Science, Israel

Abstract

This research aims to address the issue of exponential rule generation in fuzzy rule-based classification systems by developing a hybrid grid partition and rough set method. Fuzzy rule-based classification systems have the potential to construct linguistically understandable models, but a major constraint is the significant increase in the number of rules with a high number of attributes, which can diminish interpretation and classification accuracy. In this study, the grid partition method is utilized to generate fuzzy rules with adaptively adjusted grid structures, thus avoiding exponential rule proliferation. The research encompasses the use of the Iris Flower dataset, rule formation while considering variable precision, and classification accuracy testing. The research findings indicate that the hybrid grid partition and rough set method produces more efficient and accurate fuzzy rules, with a classification accuracy rate of 83.33%. This method also successfully reduces the number of generated rules, making it a promising solution to tackle the issue of exponential rule increase in fuzzy rule-based classification systems.

Corresponding Author:

Dodavah Tesseman Chislon,
Faculty of Mathematics and Computer Science,
Weizmann Institute of Science,
234 Herzl Street, POB 26, Rehovot 7610001, Israel.
Email: dodavahchislon@gmail.com.

Article Info

Article history:

Received : Jan 15, 2018;
Revised : Jun 23, 2018
accepted : May 14, 2019

Keywords:

Classification Accuracy;
Fuzzy Rule-Based Classification;
Hybrid Grid Partition;
Interpretability;
Rough Set.

This is an open access article under
the [CC BY](https://creativecommons.org/licenses/by/4.0/) license.



Introduction

Fuzzy rule-based classification systems are very useful systems in the field of machine learning because they are capable of building understandable linguistic models (Soua, et al. 2012). Fuzzy rule-based classification systems are also capable of building linguistic interpretable models that automatically generate if-then fuzzy rules and are used to classify new observations. However, in monitoring the learning system, the large number of predictive attributes still leads to an exponential increase in the number of generated rules. In addition, the antecedent conditions of the rules obtained are very large because they contain all the attributes that describe the sample. Therefore the accuracy of system interpretation also decreases (Borgi, 2018).

Many methods have been proposed to reduce the number of rules and to remove various types of rules such as irrelevant, redundant, and incorrect rules. Some of the methods that have been applied are the method of reducing the number of rules by forgetting the weak, namely the method of selecting rules based on genetic algorithms (GA), the GA method which is extended to the case of selecting rules for two objectives explicitly by considering the trade-off between the number of fuzzy if-then rules. and classification accuracy, the rule selection method by including minimization of the

total length of the rule (that is, the total number of antecedent conditions), the genetic fuzzy system (GFS) method which uses feature selection as well as the rule selection where the GFS is coupled with a learning process based on genetic algorithms, Nozaki et al. (1896); Ishibuchi et al. (1997); Ishibuchi et al. (1999); Ishibuchi & Nakashima (2001), Ishibuchi & Yamamoto (2004); Gacto et al. (2007); Herrera (2008); Akhand & Murase (2015); Borgie (2018), Dutu et al. (2018)).

Several approaches to earning fuzzy rules known as approach fuzzy grids partition type. Fuzzy rules are generated by partitioning the training data into fuzzy subsets using membership functions and then building fuzzy rules for each fuzzy grid (Ishibuchi, et al. (1992); (Ishibuchi, et al. (1993); (Ishibuchi & Yamamoto, 2004); (Ishibuchi & Yamamoto (2005); (Ishibuchi & Nojima (2007))

Soua et al. (2009) and Borgie (2018) developed a fuzzy rule-based classification system with a grid-type fuzzy partition approach to handle high-dimensional pattern classification problems. The experimental results show that the effectiveness of classification ability and interpretability are very satisfactory in classification problems for some low-dimensional patterns. However, this is not the case with high-dimensional data, the problem of increasing the number of rules still exists when the number of attributes is high.

Some studies that generate rules *fuzzy* with a grid approach, among others: Ishibuchi et al. (1992) with a grid partition approach, a distributed representation technique. R. Chen & Hu (2003) with a simple fuzzy grid approach to find fuzzy sequence patterns using a simple fuzzy partition method. Chen et al. (2016) generate several rules using a simple grid partition, then change the weights of the rules using the particle swarm optimization algorithm. Sitompul et al. (2017) using an adaptive distributed grid-partition approach, where the research produces a smaller number of rules compared to distributed grid-partition, applied the Quantum-inspired Quantum-behaved Particle Swarm Optimization (QiQPSO) algorithm to build an initial fuzzy classification system and a grid method to partition the feature space, and then the fuzzy rule base was further optimized by QiQPSO to reduce the number of fuzzy rules thereby increasing the interpretability of more classification accuracy. tall.

Interpretability is one of the desired characteristics in various classification tasks. Rule-based systems and fuzzy logic can be used for interpretation in classification but the main drawback of rule-based systems is that the complex and large rules for classification sometimes become very difficult in interpretation. Rule reduction is also difficult for various reasons and removing important rules can impact classification accuracy. Utilizes a combination of rough sets and fuzzy logic as a Rough Set-Based Hebbian Rule Reduction (RS-HeRR) method to produce effective rule sets. The proposed method leverages rule reduction through partial dependency reduction as well as system performance improvement to significantly reduce redundancy issues and even provide the same or better accuracy.

Theory *rough set* has an important role in data mining (Feng, et al. 2014), first introduced by Pawlak in 1982 (Pawlak, 1982). Rough set is a mathematical method for Artificial Intelligence (AI) applications in dealing with problems of imprecision, uncertainty or incomplete information.

Several studies since 2000 have applied rule-based generation technology *rough set* in various fields such as the decision problem of determining new products, manufacturing processes, machine operations and ordinal predictions, customer service strategies, power system stabilizers (Mak & Munakata (2002); Hou & Huang (2004); Pattaraintakorn et al. (2006); Huang et al. (2013); Anitha & Venkatesan (2014); Jia et al. (2015); Meng & Lu (2016); Fetouh & Zaky (2017)).

The researchers used various methods including; Variable Precision Rough Set (VPRS), RClass-Plus, Rough-Fuzzy Multilayer Perceptron (MLP), rough set feature selection, Cumulative Probability Distribution Approach (CPDA), Rule Extracting Algorithm Based on The Converse Approximation (REBCA), incremental rough set, Rule Generation based on Classification Attribute (RGCA) (as can be seen in Beynon et al. (2000); Khoo & Zhai (2001); Pal et al. (2003); Wang et al.

(2006);Inuiguchi & Miyajima (2007);Teoh et al. (2008);Qian et al. (2008);Fan, YN, et al. (2008);Ma, T et al. (2009)).

The hybrid method can effectively reduce the number of rules. Kumar & Yadav (2015) states that the problem of imprecise and uncertain data sets can be handled by several implementations of rough set techniques with fuzzy logic, by combining the advantages of fuzzy sets and rough sets can be used to classify objects into their respective classes. Combines genetic algorithms and intuitive fuzzy rough sets to reduce attribute sets on large-scale intuitive fuzzy information systems, and uses classes similar to intuitive fuzzy sets to extract rules on a large scale, and obtains a rule base with minimal size and configuration generation time and storage space optimal. The rough set technique is then also used together with other techniques such as data mining (Luo & Zhong 2010), genetic algorithms (Othman, et al. 2011), association rule mining (Shi, et al. 2012), rough-set believe rule model using multinomial subjective logic (Jia, et al. 2015). Lin et al. (2018) developed a fuzzy rule generation model by combining the Kernel Intuitionistic Fuzzy Means Clustering (KIFCM) method and RST as the Kernel Intuitionistic Fuzzy Rough Set Model (KIFRS) method. Proposed the integration of fuzzy logic into the application of the Functional Resonance Analysis Method (FRAM) to evaluate deicing operations from an aircraft systemic perspective. The process of integrating fuzzy logic with a high number of input variables results in a large number of rules. To further improve this proposal, the rough set method is used as a data mining tool to generate and reduce the number of rules in classifying results.

Research on solving the problem of exponentially increasing number of generated rules in fuzzy rule-based systems to handle classification problems. A hybrid grid partition and rough set technique generates fuzzy rules in the new method. The adaptive distributed fuzzy grid partition method adapts the number of rules to the number of partitions. This adaptive technique hopes to avoid an exponential rise of rules, which would affect classification accuracy, interpretability, and system.

Methods

Description of Research Architecture

Input Datasets

The dataset used for training data is the Iris Flower Dataset in table 1 which consists of 4 condition attributes, namely: sepal length (sl), sepal width (sw), petal length (pl), and petal width [pw] and 3 decision class attributes namely: setosa, versicolor, and virginica. The number of datasets inputted is 150 data:

Table 1. Iris Flower Dataset

No.	<i>sl</i>	<i>sw</i>	<i>pl</i>	<i>pw</i>	<i>Class</i>
1	5.1	3.5	1.4	0.2	<i>setosa</i>
2	4.9	3.0	1.4	0.2	<i>setosa</i>
3	4.7	3.2	1.3	0.2	<i>setosa</i>
4	4.6	3.1	1.5	0.2	<i>setosa</i>
5	5.0	3.6	1.4	0.2	<i>setosa</i>
6	5.4	3.9	1.7	0.4	<i>setosa</i>
7	4.6	3.4	1.4	0.3	<i>setosa</i>
8	5.0	3.4	1.5	0.2	<i>setosa</i>
9	4.4	2.9	1.4	0.2	<i>setosa</i>
10	4.9	3.1	1.5	0.1	<i>setosa</i>
11	5.4	3.7	1.5	0.2	<i>setosa</i>
12	4.8	3.4	1.6	0.2	<i>setosa</i>
13	4.8	3.0	1.4	0.1	<i>setosa</i>
14	4.3	3.0	1.1	0.1	<i>setosa</i>
15	5.8	4.0	1.2	0.2	<i>setosa</i>
16	5.7	4.4	1.5	0.4	<i>setosa</i>
17	5.4	3.9	1.3	0.4	<i>setosa</i>

No.	<i>sl</i>	<i>sw</i>	<i>pl</i>	<i>pw</i>	<i>Class</i>
18	5.1	3.5	1.4	0.3	<i>setosa</i>
19	5.7	3.8	1.7	0.3	<i>setosa</i>
20	5.1	3.8	1.5	0.3	<i>setosa</i>
21	5.4	3.4	1.7	0.2	<i>setosa</i>
22	5.1	3.7	1.8	0.4	<i>setosa</i>
23	4.6	3.6	1.0	0.2	<i>setosa</i>
24	5.1	3.3	1.7	0.5	<i>setosa</i>
25	4.8	3.4	1.9	0.2	<i>setosa</i>
26	5.0	3.0	1.6	0.2	<i>setosa</i>
27	5.0	3.4	1.6	0.4	<i>setosa</i>
28	5.2	3.5	1.5	0.2	<i>setosa</i>
29	5.2	3.4	1.4	0.2	<i>setosa</i>
30	4.7	3.2	1.6	0.2	<i>setosa</i>
Etc
150	5.9	3.0	5.1	1.8	<i>virginica</i>

Preprocessing Datasets

a. Data normalization

Normalizing the data set values is done by changing the actual data set value intervals into the interval [0,1]. To get a new *x* value (normalized value) is calculated according to equation 1 below:

$$Value_{New} = \frac{\text{Smallest data values}}{\text{largest data value} - \text{smallest data value}} \quad (1)$$

The same way is done for all data, the normalized Iris Flower dataset can be seen in Table 2

Table 2. Normalized Iris Flower Dataset

No.	<i>sl</i>	<i>sw</i>	<i>pl</i>	<i>pw</i>	<i>Class</i>
1	0.2222	0.6250	0.0678	0.0417	<i>setosa</i>
2	0.1667	0.4167	0.0678	0.0417	<i>setosa</i>
3	0.1111	0.5000	0.0508	0.0417	<i>setosa</i>
4	0.0833	0.4583	0.0847	0.0417	<i>setosa</i>
5	0.1944	0.6667	0.0678	0.0417	<i>setosa</i>
6	0.3056	0.7917	0.1186	0.1250	<i>setosa</i>
7	0.0833	0.5833	0.0678	0.0833	<i>setosa</i>
8	0.1944	0.5833	0.0847	0.0417	<i>setosa</i>
9	0.0278	0.3750	0.0678	0.0417	<i>setosa</i>
10	0.1667	0.4583	0.0847	0.0000	<i>setosa</i>
11	0.3056	0.7083	0.0847	0.0417	<i>setosa</i>
12	0.1389	0.5833	0.1017	0.0417	<i>setosa</i>
13	0.1389	0.4167	0.0678	0.0000	<i>setosa</i>
14	0.0000	0.4167	0.0169	0.0000	<i>setosa</i>
15	0.4167	0.8333	0.0339	0.0417	<i>setosa</i>
16	0.3889	1,0000	0.0847	0.1250	<i>setosa</i>
17	0.3056	0.7917	0.0508	0.1250	<i>setosa</i>
18	0.2222	0.6250	0.0678	0.0833	<i>setosa</i>
19	0.3889	0.7500	0.1186	0.0833	<i>setosa</i>
20	0.2222	0.7500	0.0847	0.0833	<i>setosa</i>
21	0.3056	0.5833	0.1186	0.0417	<i>setosa</i>
22	0.2222	0.7083	0.1356	0.1250	<i>setosa</i>
23	0.0833	0.6667	0.0000	0.0417	<i>setosa</i>
24	0.2222	0.5417	0.1186	0.1667	<i>setosa</i>
25	0.1389	0.5833	0.1525	0.0417	<i>setosa</i>
26	0.1944	0.4167	0.1017	0.0417	<i>setosa</i>
27	0.1944	0.5833	0.1017	0.1250	<i>setosa</i>
28	0.2500	0.6250	0.0847	0.0417	<i>setosa</i>
29	0.2500	0.5833	0.0678	0.0417	<i>setosa</i>
30	0.1111	0.5000	0.1017	0.0417	<i>setosa</i>
etc

No.	<i>sl</i>	<i>sw</i>	<i>pl</i>	<i>pw</i>	<i>Class</i>
150	0.4444	0.4167	0.6949	0.7083	virginica

b. Data transformation

Data transformation can be carried out on condition attributes and decision attributes. The only attributes that were transformed in this study were condition attributes, while decision attributes were not transformed because the data was already in categorical form. Condition attributes are transformed into several set categories. The transformation of the sepal length and petal length condition attributes is divided into three set categories, namely:

Table 3. decision attributes sepal length and petal length

No	Category	Weight
1	Short set (SS)	≤ 0.3333
2	Medium set (MS)	$0.3333 < MS \leq 0.6667$
3	Long set (LS)	> 0.6667

Meanwhile, the transformation of the sepal width and petal width attributes is divided into three categories of fuzzy sets, namely:

Table 4. decision attributes sepal width dan petal width

No	Category	Weight
1	Small Category (SC)	≤ 0.3333
2	Medium Category (MC)	$0.3333 < MC \leq 0.6667$
3	Width Category (WC)	> 0.6667

Formation of Rules

Formation of rules is the initial process in a classification system to deal with problems of imprecision, uncertainty or incompleteness of information. A large number of attributes will also cause an exponential increase in the number of rules causing interpretability or classification accuracy to decrease. Forming rules using the rough set method is to obtain a short rule estimate from an information table. The Variable Precision Rough Set (VPRS) model introduced by Ziarko (Ziarko, 1993) is used to analyze classification error as a precision parameter β (beta). The β value is defined as the classification error and ranges in the value $0 \leq \beta < 0.5$.

a. Information Systems and Decision Systems

The Information System and Decision System are a set of data consisting of a set of conditional attributes and a set of additional attributes which are targets or decisions. Data in the form of Information Systems and Decision Systems is obtained from the dataset preprocessing process. The following excerpts 9 data records of the Iris Flower dataset from table 3.4 with record numbers 9, 21, 42, 73, 93, 94, 107, 115 and 117 which are used as sample objects in the formation of rules. Data representation in the form of Information System and Decision System training data tables can be seen in table 5 below:

Table 5. Information Systems and Decision Systems

No. Data Record	Object	a	b	c	d	Class
9.	X_1	SS	MC	SS	SC	<i>setosa</i>
21.	X_2	SS	MC	SS	SC	<i>setosa</i>
42.	X_3	SS	SC	SS	SC	<i>setosa</i>
73.	X_4	MS	SC	MS	MC	<i>versicolor</i>
93.	X_5	SS	SC	MS	MC	<i>versicolor</i>
94.	X_6	SS	SC	MS	MC	<i>versicolor</i>
107.	X_7	SS	SC	MS	MC	<i>virginica</i>
115.	X_8	MS	MC	LS	WC	<i>virginica</i>
117.	X_9	MS	MC	LS	WC	<i>virginica</i>

b. Equivalence Class and Decision Class

Equivalence Class is grouping data objects that have the same value on conditional attributes while Decision Class is grouping the same class attribute values. The grouping results in the Equivalence Class and Decision Class will be used to find the precision parameter value (β). Based on Table 5, the following Equivalence Class is obtained in table 6 and Decision Class in table 7:

Table 6. Information System and Decision System

Equivalence Class	Class Members	Number of Objects
E1	X_1, X_2	2
E2	X_3	1
E3	X_4	1
E4	X_5, X_6, X_7	3
E5	X_8, X_9	2

Table 7. Decision Class

Notation	Class Members	Number of Objects
D1	X_1, X_2, X_3	3
D2	X_4, X_5, X_6	3
D3	X_7, X_8, X_9	3

c. Precision Parameter Value (β)

Determining the precision parameter (β) is by comparing the number of members in a decision class which are also part of the class members of the equivalence class divided by the number of members of an equivalence class. The results of the comparison of relationship degrees for each equivalence class are summarized in table 8 below:

Table 8. Degree of relationship

Equivalence Class	Decision		
	D1	D2	D3
E1	0	1	1
E2	0	1	1
E3	1	0	1
E4	1	1/3	2/3
E5	1	1	0

d. Set β – reduct

It is known that the precision value of β is 1/3, then the set of β -reducts can be determined by looking at the location of 1/3, namely in the Equivalence row E4 and in column D2. The β -reduct set ($C\beta$), namely:

$$C\beta = E4 = \{X_5, X_6, X_7\} \dots \dots \dots (2)$$

To obtain a subset of minimal attributes as a whole, Discernibility Matrix Modulo D is used to compare the contents of an object's attributes with other objects. The attributes compared are the condition attributes and the decision attributes. If the attribute values are the same then it will not produce a value, but if the attribute values being compared are different then it will produce a value. Discernibility Matrix Modulo D for β -reduct can be seen in table 9 below.

Table 9. Discernibility Matrix Modulo D for β -reduct

	X_1	X_2	X_3	X_4	X_5	X_6	X_7	X_8	X_9
X_1	x	x	x	a B C D	bcd	bcd	bcd	acd	acd
X_2	x	x	x	a B C D	bcd	bcd	bcd	acd	acd
X_3	x	x	x	acd	CD	CD	CD	a B C D	a B C D
X_4	a B C D	a B C D	acd	x	x	x	a	bcd	bcd
X_5	bcd	bcd	CD	x	x	x	x	a B C D	a B C D

X_6	bcd	bcd	CD	x	x	x	x	a B C D	a B C D
X_7	bcd	bcd	CD	a	x	x	x	x	x
X_8	acd	acd	a B C D	bcd	a B C D	a B C D	x	x	x
X_9	acd	acd	a B C D	bcd	a B C D	a B C D	x	x	x

Rule Generation

Rule generation applies the adaptive fuzzy grid partition method where the grid structure is adaptively formed based on rule weights. Rule generation is done with 2 phases. The first phase will form a grid as smooth as possible with the provision that all possible rules can be generated, or the iteration will stop when there is a rule that has a weight of one. While in the second phase a new rule will be generated with a higher weight to increase the accuracy of the rule. The steps for generating rules with the fuzzy grid partition method are as follows:

- a. Determine Center Point and Distance from Center Point

For the number of partitions $K = 2$ and $i = 1$ then, a_1^2 is the center point for each triangular function and b^2 is the distance from the center point to the legs of the triangular function.

For a_1^2 and b^2 are calculated as follows:

$$a_i^K = \frac{i - 1}{K - 1}, i = 1, 2, 3, \dots, K \dots\dots\dots (3)$$

- b. Determining Degree of Membership (μ)

The degree of membership for each attribute on the object will be determined by the following equation 2.

$$\mu_i^K(x) = \frac{b - x}{b - a} \dots\dots\dots (4)$$

Calculation of μ on $K \leftarrow 2$ for the data set with the sepal length (sl) attribute with data record number 10 as follows:

Table 10. Degree of membership in $K \leftarrow 2$

Class	No. Data Record	μ	Membership Degree		
			sl	sw	pw
Setosa	9	μ_1^2	0.9722	0.6250	0.9583
		μ_2^2	0.0278	0.3750	0.0417
	42	μ_1^2	0.9444	0.8750	0.9167
		μ_2^2	0.0556	0.1250	0.0833
Versicolor	73	μ_1^2	0.4444	0.7917	0.4167
		μ_2^2	0.5556	0.2083	0.5833
	93	μ_1^2	0.5833	0.7500	0.5417
		μ_2^2	0.4167	0.2500	0.4583
virginica	107	μ_1^2	0.8333	0.7917	0.3333
		μ_2^2	0.1667	0.2083	0.6667
	115	μ_1^2	0.5833	0.6667	0.0417
		μ_2^2	0.4167	0.3333	0.9583

- c. Calculation of α -predicate

The α -predicate is calculated on each class against all rules that have the potential to be generated. The α -predicate calculation is performed on the Setosa data set rule R_{111}^2 with the following equation:

$$R_{111}^2 = \mu_1^2(sl) \cdot \mu_1^2(sw) \cdot \mu_1^2(pw) \dots\dots\dots (5)$$

- d. Determination of β

β is the largest membership degree value of the α -predicate values in each candidate rule in the same class. The determination of β is done by comparing all α -predicates in the same candidate rule. In the candidate rule R_{111}^2 in the versicolor class, it is done by comparing the α -predicate values in the following equation:

$$\beta = \max \{ \alpha - \text{predicate } R_{111}^2(\text{data set}) ; \alpha - \text{predicate } R_{111}^2(\text{data set}) \} \dots\dots (6)$$

e. Determination of Conclusions

The conclusion of the rule is determined based on the largest β value among β_{setosa} , $\beta_{\text{versicolor}}$, and $\beta_{\text{virginica}}$, which is called β_{max} . If there are two or more β_{max} that have the same value, or the β_{max} value is 0, then the conclusion cannot be determined and the rule cannot be generated. The value of β_{max} for R_{111}^2 is determined by the following equation:

$$\beta_{\text{max}} = \max \{ \beta_{\text{setosa}}, \beta_{\text{versicolor}}, \beta_{\text{virginica}} \} \quad \dots\dots\dots (7)$$

f. Rule Weight Calculation

The weight of each rule is calculated using the following equation:

$$CF_{ij}^K = \frac{|\beta_{\text{max}} - (\beta_1 + \beta_2)/2|}{\beta_{\text{max}} + \beta_1 + \beta_2} \quad \dots\dots\dots (8)$$

where:

$$CF_{ij}^K = \text{rule weight label}, i = 1, 2, \dots, K, j = 1, 2, \dots, K$$

$$\beta_{\text{max}} = \text{value } \beta \text{ largest among } \beta_{\text{setosa}}, \beta_{\text{versicolor}}, \text{ and } \beta_{\text{virginica}}.$$

$$\beta_1, \beta_2 = \text{value } \beta \text{ other than } \beta_{\text{max}}.$$

g. Rule Generation for $K = 3$

For $K=3$, there are 27 potential rules to be generated obtained from K^d , where d is the number of dataset attributes.

h. Rule Generation for $K = 2K$

For $K=2K = 4$, there are 64 potential rules to be generated obtained from K^d , where d is the number of dataset attributes.

Adapted Partition Grid

The rule weights determine the grid structure formed by the modified grid division approach. Rule generation is two-stage. The grid will be built as smoothly as feasible in the first stage if all conceivable rules can be generated or the iteration stops when a rule with a weight of one is found. To improve rule accuracy, a new rule with a higher weight will be developed in the second step.

The steps of the adapted grid partition method for obtaining rules are obtained by means of:

1. Determine $K = 2$, $K_{\text{stage1}} = 0$, $SR = []$ and stage = 1.
2. Calculate the μ value according to the K value.
3. Calculate the α -predicate based on the μ value in step b.
4. Calculate the value of β in each class.
5. Calculate β_{max} to determine the conclusions.
6. Calculate the CF value of each rule.
 - (a.) If in step 1, empty the SR if it is not empty, then add the rules that can be generated to the SR.
 - (b.) If in stage 2, add the rules that can be generated to the SR.
7. Check the stopping criteria, if met go to step p.
8. If the value is in stage 2, go to step m
9. (a.) If all potential rules can be generated and there is a CF value of 1, stage \leftarrow stage + 1, $K_{\text{stage1}} \leftarrow K$, continue the mth step.
 - (b.) If there are rules that cannot be generated ($\beta_{\text{max}} = 0$), Stage \leftarrow Stage + 1, continue step l.
10. For $K \leftarrow K + 1$. return to step b.
11. Empty SR if not empty, $K \leftarrow K - 1$, $K_{\text{stage1}} \leftarrow K$, and return to step b.
12. Select the rule on the SR with the lowest CF value.

13. Initialize K and K_{tahap1} with the value of the rule R_{ijk}^K selected in step m then $K \leftarrow 2K$, if $K_{\text{tahap1}} \neq K$, remove the rule R_{ijk}^K selected in step n from the SR.
14. Return to step b.
15. Stop.

Testing Method

Testing the method is done by classifying objects on the dataset against the generated rules. The steps in performing the classification process are as follows:

- a. Calculating the α -value of each setosa class, versicolor class and virginica class. The α value is the result of multiplying the α -predicate by the rule weight (CF). The α value is calculated with the following equation:

$$\alpha = \mu_i^K (sl) \cdot \mu_j^K (sw) \cdot \mu_k^K (pw) \cdot CF_{ijk}^K$$

- b. Determine the largest α value in each class α_{setosa} , $\alpha_{\text{versicolor}}$ and $\alpha_{\text{virginica}}$.
- c. Determine the largest α_{max} value of each class for each data record with the equation:

$$\alpha_{\text{max}} = \max \{ \alpha_{\text{setosa}}, \alpha_{\text{versicolor}}, \alpha_{\text{virginica}} \}$$

Results and Discussion

Dataset Preprocessing Results

The initial step of the research is to prepare the dataset to be processed using the proposed method. The dataset used is the Iris flower dataset obtained from the UCI Machine Learning Repository with a total of 150 objects and a total of 4 attributes. The dataset is normalized, then categorization is performed for each attribute. Attribute a is sepal length (sl), attribute b is sepal width (sw), attribute c is petal length (pl) and attribute d is petal width (pw). The results of normalization and categorization are information systems and decision systems as shown in Table 11 below:

Table 11. Information system and decision system Preprocessing

Objek	a	b	c	d	Class
X_1	SS	MC	SS	SC	setosa
X_2	SS	MC	SS	SC	setosa
X_3	SS	MC	SS	SC	setosa
X_4	SS	MC	SS	SC	setosa
X_5	SS	MC	SS	SC	setosa
X_6	SS	WC	SS	SC	setosa
X_7	SS	MC	SS	SC	setosa
X_8	SS	MC	SS	SC	setosa
X_9	SS	MC	SS	SC	setosa
X_{10}	SS	MC	SS	SC	setosa
X_{11}	SS	WC	SS	SC	setosa
X_{12}	SS	MC	SS	SC	setosa
X_{13}	SS	MC	SS	SC	setosa
X_{14}	SS	MC	SS	SC	setosa
X_{15}	MS	WC	SS	SC	setosa
X_{16}	MS	WC	SS	SC	setosa
X_{17}	SS	WC	SS	SC	setosa
X_{18}	SS	MC	SS	SC	setosa
X_{19}	MS	WC	SS	SC	setosa
X_{20}	SS	WC	SS	SC	setosa
X_{21}	SS	MC	SS	SC	setosa
X_{22}	SS	WC	SS	SC	setosa
X_{23}	SS	MC	SS	SC	setosa
X_{24}	SS	MC	SS	SC	setosa
X_{25}	SS	MC	SS	SC	setosa
X_{26}	SS	MC	SS	SC	setosa
X_{27}	SS	MC	SS	SC	setosa

Objek	<i>a</i>	<i>b</i>	<i>c</i>	<i>d</i>	Class
X_{28}	SS	MC	SS	SC	<i>setosa</i>
X_{29}	SS	MC	SS	SC	<i>setosa</i>
X_{30}	SS	MC	SS	SC	<i>setosa</i>
X_{31}	SS	MC	SS	SC	<i>setosa</i>
X_{32}	SS	MC	SS	SC	<i>setosa</i>
X_{33}	SS	WC	SS	SC	<i>setosa</i>
X_{34}	MS	WC	SS	SC	<i>setosa</i>
X_{35}	SS	MC	SS	SC	<i>setosa</i>
X_{36}	SS	MC	SS	SC	<i>setosa</i>
X_{37}	MS	MC	SS	SC	<i>setosa</i>
X_{38}	SS	MC	SS	SC	<i>setosa</i>
X_{39}	SS	MC	SS	SC	<i>setosa</i>
X_{40}	SS	MC	SS	SC	<i>setosa</i>
X_{41}	SS	MC	SS	SC	<i>setosa</i>
X_{42}	SS	SC	SS	SC	<i>setosa</i>
X_{43}	SS	MC	SS	SC	<i>setosa</i>
X_{44}	SS	MC	SS	SC	<i>setosa</i>
X_{45}	SS	WC	SS	SC	<i>setosa</i>
X_{46}	SS	MC	SS	SC	<i>setosa</i>
X_{47}	SS	WC	SS	SC	<i>setosa</i>
X_{48}	SS	MC	SS	SC	<i>setosa</i>
X_{49}	SS	WC	SS	SC	<i>setosa</i>
X_{50}	SS	MC	SS	SC	<i>setosa</i>
X_{51}	SS	MC	MS	MC	<i>versicolor</i>
X_{52}	MS	MC	MS	MC	<i>versicolor</i>
X_{53}	LS	MC	MS	MC	<i>versicolor</i>
X_{54}	MS	SC	MS	MC	<i>versicolor</i>
X_{55}	MS	MC	MS	MC	<i>versicolor</i>
X_{56}	MS	MC	MS	MC	<i>versicolor</i>
X_{57}	MS	MC	MS	MC	<i>versicolor</i>
X_{58}	SS	SC	MS	MC	<i>versicolor</i>
X_{59}	MS	MC	MS	MC	<i>versicolor</i>
X_{60}	SS	SC	MS	MC	<i>versicolor</i>
X_{61}	SS	SC	MS	MC	<i>versicolor</i>
X_{62}	MS	MC	MS	MC	<i>versicolor</i>
X_{63}	MS	SC	MS	MC	<i>versicolor</i>
X_{64}	MS	MC	MS	MC	<i>versicolor</i>
X_{65}	MS	MC	MS	MC	<i>versicolor</i>
X_{66}	MS	MC	MS	MC	<i>versicolor</i>
X_{67}	MS	MC	MS	MC	<i>versicolor</i>
X_{68}	MS	SC	MS	MC	<i>versicolor</i>
X_{69}	MS	SC	MS	MC	<i>versicolor</i>
X_{70}	MS	SC	MS	MC	<i>versicolor</i>
X_{71}	MS	MC	MS	WC	<i>versicolor</i>
X_{72}	MS	MC	MS	MC	<i>versicolor</i>
X_{73}	MS	SC	MS	MC	<i>versicolor</i>
X_{74}	MS	MC	MS	MC	<i>versicolor</i>
X_{75}	MS	MC	MS	MC	<i>versicolor</i>
X_{76}	MS	MC	MS	MC	<i>versicolor</i>
X_{77}	LS	MC	MS	MC	<i>versicolor</i>
X_{78}	MS	MC	LS	MC	<i>versicolor</i>
X_{79}	MS	MC	MS	MC	<i>versicolor</i>
X_{80}	MS	SC	MS	MC	<i>versicolor</i>
X_{81}	MS	SC	MS	MC	<i>versicolor</i>
X_{82}	MS	SC	MS	MC	<i>versicolor</i>
X_{83}	MS	SC	MS	MC	<i>versicolor</i>
X_{84}	MS	SC	MS	MC	<i>versicolor</i>
X_{85}	SS	MC	MS	MC	<i>versicolor</i>
X_{86}	MS	MC	MS	MC	<i>versicolor</i>

Objek	<i>a</i>	<i>b</i>	<i>c</i>	<i>d</i>	Class
X_{87}	MS	MC	MS	MC	<i>versicolor</i>
X_{88}	MS	SC	MS	MC	<i>versicolor</i>
X_{89}	MS	MC	MS	MC	<i>versicolor</i>
X_{90}	MS	SC	MS	MC	<i>versicolor</i>
X_{91}	MS	SC	MS	MC	<i>versicolor</i>
X_{92}	MS	MC	MS	MC	<i>versicolor</i>
X_{93}	MS	SC	MS	MC	<i>versicolor</i>
X_{94}	SS	SC	MS	MC	<i>versicolor</i>
X_{95}	MS	SC	MS	MC	<i>versicolor</i>
X_{96}	MS	MC	MS	MC	<i>versicolor</i>
X_{97}	MS	MC	MS	MC	<i>versicolor</i>
X_{98}	MS	MC	MS	MC	<i>versicolor</i>
X_{99}	SS	SC	MS	MC	<i>versicolor</i>
X_{100}	MS	MC	MS	MC	<i>versicolor</i>
X_{101}	MS	MC	LS	WC	<i>virginica</i>
X_{102}	MS	SC	LS	WC	<i>virginica</i>
X_{103}	LS	MC	LS	WC	<i>virginica</i>
X_{104}	MS	MC	LS	WC	<i>virginica</i>
X_{105}	MS	MC	LS	WC	<i>virginica</i>
X_{106}	LS	MC	LS	WC	<i>virginica</i>
X_{107}	SS	SC	MS	MC	<i>virginica</i>
X_{108}	LS	MC	LS	WC	<i>virginica</i>
X_{109}	MS	SC	LS	WC	<i>virginica</i>
X_{110}	LS	MC	LS	WC	<i>virginica</i>
X_{111}	MS	MC	LS	WC	<i>virginica</i>
X_{112}	MS	SC	LS	WC	<i>virginica</i>
X_{113}	LS	MC	LS	WC	<i>virginica</i>
X_{114}	MS	SC	LS	WC	<i>virginica</i>
X_{115}	MS	MC	LS	WC	<i>virginica</i>
X_{116}	MS	MC	LS	WC	<i>virginica</i>
X_{117}	MS	MC	LS	WC	<i>virginica</i>
X_{118}	LS	WC	LS	WC	<i>virginica</i>
X_{119}	LS	SC	LS	WC	<i>virginica</i>
X_{120}	MS	SC	LS	MC	<i>virginica</i>
X_{121}	LS	MC	LS	WC	<i>virginica</i>
X_{122}	MS	MC	MS	WC	<i>virginica</i>
X_{123}	LS	MC	LS	WC	<i>virginica</i>
X_{124}	MS	SC	MS	WC	<i>virginica</i>
X_{125}	MS	MC	MS	WC	<i>virginica</i>
X_{126}	LS	MC	LS	WC	<i>virginica</i>
X_{127}	MS	MC	MS	WC	<i>virginica</i>
X_{128}	MS	MC	MS	WC	<i>virginica</i>
X_{129}	MS	MC	LS	WC	<i>virginica</i>
X_{130}	LS	MC	LS	MC	<i>virginica</i>
X_{131}	LS	MC	LS	WC	<i>virginica</i>
X_{132}	LS	WC	LS	WC	<i>virginica</i>
X_{133}	MS	MC	LS	WC	<i>virginica</i>
X_{134}	MS	MC	LS	MC	<i>virginica</i>
X_{135}	MS	SC	LS	MC	<i>virginica</i>
X_{136}	LS	MC	LS	WC	<i>virginica</i>
X_{137}	MS	MC	LS	WC	<i>virginica</i>
X_{138}	MS	MC	LS	WC	<i>virginica</i>
X_{139}	MS	MC	MS	WC	<i>virginica</i>
X_{140}	LS	MC	LS	WC	<i>virginica</i>
X_{141}	MS	MC	LS	WC	<i>virginica</i>
X_{142}	LS	MC	LS	WC	<i>virginica</i>
X_{143}	MS	SC	LS	WC	<i>virginica</i>
X_{144}	LS	MC	LS	WC	<i>virginica</i>
X_{145}	MS	MC	LS	WC	<i>virginica</i>

Objek	a	b	c	d	Class
X_{146}	MS	MC	LS	WC	virginica
X_{147}	MS	SC	LS	WC	virginica
X_{148}	MS	MC	LS	WC	virginica
X_{149}	MS	MC	LS	WC	virginica
X_{150}	MS	MC	LS	WC	virginica

Rule Formation Results

Rule formation is the dataset processing stage taken from the data in table 11 before the rule generation process is carried out. The formation of the rules applies the rough set method to produce a set of reduct attributes by considering the variable precision or error rate to get the reduct attribute. Information System Reduct in table 12 is reviewed again with regard to the redundancy pattern of the condition attribute values and the resulting target attribute values. In table 12, objects x_1 and x_2 are repeating objects, so one object can be omitted. Likewise with objects x_5 with x_6 and objects x_8 with x_9 . The results of forming rules with attributes with minimum reduct frequency and the results of reducing data records number 9, 21, 42, 73, 93, 94, 107, 115 and 117 can be seen in Table 12 and Table 13 below:

Table 12. Information System Reduct

No. Record data	Object	a	b	d	Class
9	X_1	SS	MC	SC	setosa
21	X_2	SS	MC	SC	setosa
42	X_3	SS	SC	SC	setosa
73	X_4	MS	SC	MC	versicolor
93	X_5	SS	SC	MC	versicolor
94	X_6	SS	SC	MC	versicolor
107	X_7	SS	SC	MC	virginica
115	X_8	MS	MC	WC	virginica
117	X_9	MS	MC	WC	virginica

Table 13. Information System Reduction

No. Data Record	Object	a	b	d	Class
9	X_1	SS	MC	SC	setosa
42	X_3	SS	SC	SC	setosa
73	X_4	MS	SC	MC	versicolor
93	X_5	SS	SC	MC	versicolor
107	X_7	SS	SC	MC	virginica
115	X_8	MS	MC	WC	virginica

Rule Generation Results

The rule generation applies the fuzzy grid partition method in which the grid structure is built using an adapted technique based on the rule weights. The adapted fuzzy grid partition method is carried out in 2 stages. In the first stage, the grid is formed as smooth as possible with the condition that all possible rules can be generated, or the iteration will stop when there is a rule that has a weight of one. Meanwhile, in the second stage, new rules will be generated with higher weights to increase the accuracy of these rules.

In accordance with the steps of the grid partition method adapted to obtain rules, the number of rules generated is 11 rules with data record numbers 9, 42, 73, 93, 107 and 115. In the first stage, the rules generated in the Kstage1 state $\leftarrow K=2$ is $S_R = \{R_{111}^2, R_{112}^2, R_{121}^2, R_{122}^2, R_{211}^2, R_{212}^2, R_{221}^2, R_{222}^2\}$. The second stage of the rules is generated in the K state $\leftarrow 2K$, is $S_R = \{R_{111}^2, R_{112}^2, R_{121}^2, R_{122}^2, R_{211}^2, R_{212}^2, R_{221}^2, R_{222}^2, R_{313}^4, R_{323}^4, R_{324}^4\}$. The rules generated up to the second stage are as follows:

1. R_{111}^2 : IF sl is A_1^2 AND sw is A_1^2 AND pw is A_1^2 THEN setosa
2. R_{112}^2 : IF sl is A_1^2 AND sw is A_1^2 AND pw is A_2^2 THEN virginica

3. $R_{121}^2: IF\ sl\ is\ A_1^2\ AND\ sw\ is\ A_2^2\ AND\ pw\ is\ A_1^2\ THEN\ setosa$
4. $R_{122}^2: IF\ sl\ is\ A_1^2\ AND\ sw\ is\ A_2^2\ AND\ pw\ is\ A_2^2\ THEN\ virginica$
5. $R_{211}^2: IF\ sl\ is\ A_2^2\ AND\ sw\ is\ A_1^2\ AND\ pw\ is\ A_1^2\ THEN\ versicolor$
6. $R_{212}^2: IF\ sl\ is\ A_2^2\ AND\ sw\ is\ A_1^2\ AND\ pw\ is\ A_2^2\ THEN\ virginica$
7. $R_{221}^2: IF\ sl\ is\ A_2^2\ AND\ sw\ is\ A_2^2\ AND\ pw\ is\ A_1^2\ THEN\ versicolor$
8. $R_{222}^2: IF\ sl\ is\ A_2^2\ AND\ sw\ is\ A_2^2\ AND\ pw\ is\ A_2^2\ THEN\ virginica$
9. $R_{313}^4: IF\ sl\ is\ A_3^4\ AND\ sw\ is\ A_1^4\ AND\ pw\ is\ A_3^4\ THEN\ virginica$
10. $R_{323}^4: IF\ sl\ is\ A_3^4\ AND\ sw\ is\ A_3^4\ AND\ pw\ is\ A_3^4\ THEN\ virginica$
11. $R_{324}^4: IF\ sl\ is\ A_3^4\ AND\ sw\ is\ A_4^4\ AND\ pw\ is\ A_4^4\ THEN\ virginica$

Classification Results

Method testing on the dataset is carried out by looking at the accuracy of the classification results for each object from the dataset. Testing is carried out in three stages, namely, first determining the α value of each object in each class; the second determines the largest α value of each object in each class and the third determines the maximum α (α_{max}) of α_{setosa} , $\alpha_{versicolor}$ and $\alpha_{virginica}$. A recapitulation of classification results can be seen in table 14 below:

Table 14. Recapitulation of classification results based on the generated rules

No. Data Record	Max			α_{max}	Class	Classification Results
	α_{setosa}	$\alpha_{versicolor}$	$\alpha_{virginica}$			
9	0.2642	0.0085	0.0107	0.2642	setosa	Correct
42	0.3437	0.0228	0.0292	0.3437	setosa	Correct
73	0.0665	0.2698	0.0871	0.2698	versicolor	Correct
93	0.1075	0.0672	0.0850	0.1075	setosa	Wrong
107	0.0998	0.0225	0.1866	0.1866	virginica	Correct
115	0.0074	0.0059	0.2188	0.2188	virginica	Correct

Classification Accuracy

To determine the classification accuracy based on the rules that have been generated, it is necessary to calculate the percentage of classification accuracy. Classification accuracy is calculated by comparing the number of correct classifications with the overall classification. Calculation of accuracy using the following equation 15.

$$accuracy(\%) = \left(\frac{\sum_{correct\ classification}}{\sum_{overall\ classification}} \times 100 \right) \% \dots\dots\dots (9)$$

Based on table 15, the level of classification accuracy based on the rules generated is as follows:

$$accuracy(\%) = \left(\frac{5}{6} \times 100 \right) \%$$

$$accuracy(\%) = 83.3333 \%$$

To find out the level of error (error) of the rules that have been generated is to compare the number of incorrect classifications with the overall classification. Measuring the level of accuracy of the rules using the following equation.

$$error(\%) = \left(\frac{\sum_{misclassification}}{\sum_{overall\ classification}} \times 100 \right) \% \dots\dots\dots (10)$$

Based on the table of 15 error levels, the rules that have been generated are as follows:

$$error(\%) = \left(\frac{1}{6} \times 100 \right) \%$$

$$error(\%) = 16.6667 \%$$

Apart from measuring the level of accuracy of the rules, method testing also measures the amount of data that cannot be classified (unclass). Unclass data can be calculated using the following equation: Based on table 15, the amount of unclass data is as follows:

$$\sum unclass = 6 - (5 + 1)$$

$$\sum unclass = 0$$

Based on the calculation results it is known that the data that cannot be classified (unclass) is 0 which means that all data can be classified.

Discussion

a. Number of Attributes and Number of Objects

One of the objectives of hybrid research on the fuzzy grid partition method with the rough set method in generating rules for data set classification is to obtain rules that do not increase exponentially. To solve this problem, the first stage was to apply the rough set method to reduce the number of attributes and the number of objects without affecting the classification results. The results of the simplification of the Discernibility Matrix Modulo D set with the CNF Boolean Function obtain candidates for the set with a minimum frequency of reduct occurrence and non-repetition. In addition, data that has repeated (redundant) values can be reduced so that the rules formed are more concise. Table 10 shows a dataset that still has 4 attributes.

In table 11 there are objects with redundant condition attributes and decision attributes, namely X_1 with X_2 , X_5 with X_6 then X_8 with X_9 . These repeated objects can be reduced by deleting one to form a rule to get a target or decision. The reduced results of Information System Reduct can be seen in table 12 which is a Reduction Information System.

In table 10 and table 12 there are differences in the number of attributes and number of objects. The rules formed by applying the rough set method are more concise. The Information System and Decision System in table 10 consist of 4 condition attributes and 9 objects while the reduced Information System in table 12 consists of 3 condition attributes and 6 objects.

b. Number of generated rules

The fuzzy grid partition method with adapted techniques can generate rules so that the resulting rules can classify data sets. The initial stage of forming rules using the rough set method can form more concise rules so that the process of generating rules using the fuzzy grid partition method also results in fewer rules being generated. In previous research (Sitompul, et al, 2017), the application of the fuzzy grid partition method with the Iris Flower data set did not carry out the process of forming rules. The rules that have the potential to be generated with a total of 9 objects and a total of 4 attributes in stage 1 with $K=2$ are 16 rules while the formation of rules that have been carried out by applying the rough set method, where the number of objects as many as 9 turned into 6 objects and the number of attributes 4 conditions changed 3 attribute conditions, then in stage 1 where $K = 2$ produces 8 rules that have the potential to be generated. The difference in the number of rules that have the potential to be generated can be seen in the following:

1. $R_{1111}^2: IF sl \text{ is } A_1^2 \text{ AND } sw \text{ is } A_1^2 \text{ AND } pl \text{ is } A_1^2 \text{ AND } pw \text{ is } A_1^2 \text{ THEN } C$
2. $R_{1112}^2: IF sl \text{ is } A_1^2 \text{ AND } sw \text{ is } A_1^2 \text{ AND } pl \text{ is } A_1^2 \text{ AND } pw \text{ is } A_2^2 \text{ THEN } C$
3. $R_{1121}^2: IF sl \text{ is } A_1^2 \text{ AND } sw \text{ is } A_1^2 \text{ AND } pl \text{ is } A_2^2 \text{ AND } pw \text{ is } A_1^2 \text{ THEN } C$
4. $R_{1122}^2: IF sl \text{ is } A_1^2 \text{ AND } sw \text{ is } A_1^2 \text{ AND } pl \text{ is } A_2^2 \text{ AND } pw \text{ is } A_2^2 \text{ THEN } C$
5. $R_{1211}^2: IF sl \text{ is } A_1^2 \text{ AND } sw \text{ is } A_2^2 \text{ AND } pl \text{ is } A_1^2 \text{ AND } pw \text{ is } A_1^2 \text{ THEN } C$
6. $R_{1212}^2: IF sl \text{ is } A_1^2 \text{ AND } sw \text{ is } A_2^2 \text{ AND } pl \text{ is } A_1^2 \text{ AND } pw \text{ is } A_2^2 \text{ THEN } C$
7. $R_{1221}^2: IF sl \text{ is } A_1^2 \text{ AND } sw \text{ is } A_2^2 \text{ AND } pl \text{ is } A_2^2 \text{ AND } pw \text{ is } A_1^2 \text{ THEN } C$
8. $R_{1222}^2: IF sl \text{ is } A_1^2 \text{ AND } sw \text{ is } A_2^2 \text{ AND } pl \text{ is } A_2^2 \text{ AND } pw \text{ is } A_2^2 \text{ THEN } C$
9. $R_{2111}^2: IF sl \text{ is } A_2^2 \text{ AND } sw \text{ is } A_1^2 \text{ AND } pl \text{ is } A_1^2 \text{ AND } pw \text{ is } A_1^2 \text{ THEN } C$
10. $R_{2112}^2: IF sl \text{ is } A_2^2 \text{ AND } sw \text{ is } A_1^2 \text{ AND } pl \text{ is } A_1^2 \text{ AND } pw \text{ is } A_2^2 \text{ THEN } C$
11. $R_{2121}^2: IF sl \text{ is } A_2^2 \text{ AND } sw \text{ is } A_1^2 \text{ AND } pl \text{ is } A_1^2 \text{ AND } pw \text{ is } A_2^2 \text{ THEN } C$
12. $R_{2122}^2: IF sl \text{ is } A_2^2 \text{ AND } sw \text{ is } A_1^2 \text{ AND } pl \text{ is } A_2^2 \text{ AND } pw \text{ is } A_2^2 \text{ THEN } C$
13. $R_{2211}^2: IF sl \text{ is } A_2^2 \text{ AND } sw \text{ is } A_2^2 \text{ AND } pl \text{ is } A_1^2 \text{ AND } pw \text{ is } A_1^2 \text{ THEN } C$

14. $R_{2212}^2: IF\ sl\ is\ A_2^2\ AND\ sw\ is\ A_2^2\ AND\ pl\ is\ A_1^2\ AND\ pw\ is\ A_2^2\ THEN\ C$
 15. $R_{2221}^2: IF\ sl\ is\ A_2^2\ AND\ sw\ is\ A_2^2\ AND\ pl\ is\ A_2^2\ AND\ pw\ is\ A_1^2\ THEN\ C$
 16. $R_{2222}^2: IF\ sl\ is\ A_2^2\ AND\ sw\ is\ A_2^2\ AND\ pl\ is\ A_2^2\ AND\ pw\ is\ A_2^2\ THEN\ C$

Figure 1. Potentially generated rules without rule generation

1. $R_{111}^2: IF\ sl\ is\ A_1^2\ AND\ sw\ is\ A_1^2\ AND\ pw\ is\ A_1^2\ THEN\ C$
 2. $R_{112}^2: IF\ sl\ is\ A_1^2\ AND\ sw\ is\ A_1^2\ AND\ pw\ is\ A_2^2\ THEN\ C$
 3. $R_{121}^2: IF\ sl\ is\ A_1^2\ AND\ sw\ is\ A_2^2\ AND\ pw\ is\ A_1^2\ THEN\ C$
 4. $R_{122}^2: IF\ sl\ is\ A_1^2\ AND\ sw\ is\ A_2^2\ AND\ pw\ is\ A_2^2\ THEN\ C$
 5. $R_{211}^2: IF\ sl\ is\ A_2^2\ AND\ sw\ is\ A_1^2\ AND\ pw\ is\ A_1^2\ THEN\ C$
 6. $R_{212}^2: IF\ sl\ is\ A_2^2\ AND\ sw\ is\ A_1^2\ AND\ pw\ is\ A_2^2\ THEN\ C$
 7. $R_{221}^2: IF\ sl\ is\ A_2^2\ AND\ sw\ is\ A_2^2\ AND\ pw\ is\ A_1^2\ THEN\ C$
 8. $R_{222}^2: IF\ sl\ is\ A_2^2\ AND\ sw\ is\ A_2^2\ AND\ pw\ is\ A_2^2\ THEN\ C$

Figure 2. Potentially generated rules with rule generation

Likewise for $K \leftarrow K+1$ and $K \leftarrow 2K$, the number of rules that have the potential to be generated is less than the fuzzy grid partition method without rule formation. When $K \leftarrow K+1$, the number of rules that can potentially be generated is 27 rules and when $K \leftarrow 2K$, the number of rules that have the potential to be generated is 64 rules.

The hybrid fuzzy grid partition method with adapted techniques and the rough set method produces 11 rules. The number of rules produced is different from the number of rules produced in previous research, namely 22 rules. The difference in the number of rules generated can be seen in table 15 below:

Table 15. The difference in the number of generated rules

Method	Original Number of Objects	The number of objects after the rough set process	Number of attributes generated	Number of Rules raised				
				K2←	KK+1←	K2K←	Stage1	Stage 2
<i>Fuzzy grid partition</i>	9	9	4	16	28	23	16	28
<i>Hybridfuzzy grid partition and rough set methods</i>	9	6	3	8	16	23	8	11

c. Classification Accuracy Results

Testing the method for classification accuracy based on the rules generated by the data in table 14 produces an accuracy rate of 83.3%. The percentage level of classification accuracy is higher than the application of the fuzzy grid partition method alone with the same amount of recorded data.

In addition to calculating the level of classification accuracy, data that cannot be classified (unclassified) is also calculated using equation 16. Based on table 16, the amount of unclassified data is as follows:

$$\sum unclass = \sum overall\ classification - \left(\sum correct\ classification + \sum incorrect\ classification \right)$$

$$\sum unclass = 6 - (6 + 2)$$

$$\sum unclass = 0$$

Based on the results of these calculations, the data that cannot be classified (unclass) is 0 which means that all data can be classified.

Conclusion

The conclusions of this research can be described based on the findings, discussions and results above are: The application of the rough set method at the beginning of rule formation can reduce the number of condition attributes and the number of redundant objects so that the rule formation process becomes more concise. The grid partition method with a grid structure applying adapted techniques produces fuzzy rules that have the potential to be generated. The hybrid grid partition method and rough set method produce classification rules that do not increase exponentially. The number of classification rules generated decreases as the number of condition attributes and the number of objects classified decrease. Fuzzy rules generated by the hybrid method produce a classification accuracy rate of 83.3% with 9 data records and the number of unclassified data is 0.

References

- Akhand, M. A. H., & Murase, K. (2015). Optimization of Fuzzy Logic Controllers with Rule Base Size Reduction using Genetic Algorithms. *Optimization of Fuzzy Logic Controllers with Rule Base Size Reduction using Genetic Algorithms*. <https://doi.org/10.1142/S0219622015500273>
- Angelov, P. P., & Zhou, X. (2008). Evolving fuzzy-rule-based classifiers from data streams. *IEEE Transactions on Fuzzy Systems*, 16(6), 1462–1475. <https://doi.org/10.1109/TFUZZ.2008.925904>
- Anitha, K., & Venkatesan, P. (2014). Rough Set Theory Approach to Generating Classification Rules. 4(3), 229–235.
- Antonelli, M., Ducange, P., & Marcelloni, F. (2014). A fast and efficient multi-objective evolutionary learning scheme for fuzzy rule-based classifiers. *Information Sciences*, 283(June), 36–54. <https://doi.org/10.1016/j.ins.2014.06.014>
- Beynon, M., Curry, B., & Morgan, P. (2000). Classification and rule induction using rough set theory. *Expert Systems*, 17(3), 136–148.
- Borgi, A. (2018). Attributes regrouping in Fuzzy Rule Based Classification Systems : an intra-classes approach. 2018 IEEE/ACS 15th International Conference on Computer Systems and Applications (AICCSA), 1–7. IEEE.
- Borzooei, R.A., Estaji, A.A., & Mobini, M. (2017). On the category of rough sets. *Soft Computing*, 21(9), 2201–2214. <https://doi.org/10.1007/s00500-016-2135-9>
- Chen, R., & Hu, Y. (2003). A Novel Method for Discovering Fuzzy Sequential Patterns Using the Simple Fuzzy Partition Method. 54(7), 660–670.
- Chen, T., Shen, Q., Su, P., & Shang, C. (2016). Fuzzy rule weight modification with particle swarm optimization. *Soft Computing*, 20(8), 2923–2937. <https://doi.org/10.1007/s00500-015-1922-z>
- De Tré, G., & Zadrozny, S. (2015). Soft computing in database and information management. In *Springer's Handbook of Computational Intelligence*. <https://doi.org/10.1007/978-3-662-43505-2>
- Dehzangi, O., Zolghadri, M.J., Taheri, S., & Fakhrahmad, S.M. (2007). Efficient fuzzy rule generation: A new approach using data mining principles and rule weighting. *Proceedings - Fourth International Conference on Fuzzy Systems and Knowledge Discovery, FSKD 2007*, 2(Fskd), 134–139. <https://doi.org/10.1109/FSKD.2007.267>
- Dutu, L., Mauris, G., Bolon, P., Dutu, L., Mauris, G., Fast, P.B.A., ... Du, L. (2018). A Fast and Accurate Rule-Base Generation Method for Mamdani Fuzzy Systems To cite this version : HAL Id : hal-01756513 A Fast and Accurate Rule-Base Generation Method for Mamdani Fuzzy Systems.
- Elkano, M., Galar, M., Sanz, J., & Bustince, H. (2016). Fuzzy Rule-Based Classification Systems for multi-class problems using binary decomposition strategies: On the influence of n-dimensional overlapping functions in the Fuzzy Reasoning Method. *Information Sciences*, 332, 94–114. <https://doi.org/10.1016/j.ins.2015.11.006>
- Feng, H., Chen, Y., Ni, Q., & Huang, J. (2014). A new rough set based classification rule generation algorithm (RGI). *Proceedings - 2014 International Conference on Computational Science and Computational Intelligence, CSCI 2014*, 1, 380–385. <https://doi.org/10.1109/CSCI.2014.71>
- Fetouh, T., & Zaky, M.S. (2017). New approach to design SVC-based stabilizers using genetic algorithm and rough set theory. *IET Generation, Transmission and Distribution*, 11(2), 372–382. <https://doi.org/10.1049/iet-gtd.2016.0701>
- Gacto, M. J., Herrera, F., & A-fdez, J. A. (2007). A multi-objective genetic algorithm for tuning and rule selection

- to obtain accurate and compact linguistic fuzzy rule-based systems^{*'*}. 15(5), 539–557.
- Hefny, H., Ghiduk, A., Wahab, A., & Elashiry, M. (2010). Effective Method for Extracting Rules from Fuzzy Decision Trees based on Ambiguity and Classifiability. *Entropy*, 3(4), 12.
- Herrera, F. (2008). Genetic fuzzy systems: taxonomy, current research trends and prospects. 27–46. <https://doi.org/10.1007/s12065-007-0001-5>
- Hou, T.H., & Huang, C.C. (2004). Application of fuzzy logic and variable precision rough set approach in a remote monitoring manufacturing process for diagnosis rule induction. *Journal of Intelligent Manufacturing*, 15(3), 395–408. <https://doi.org/10.1023/B:JIMS.0000026576.00445.d8>
- Huang, CC, Tseng, TL, Fan, YN, & Hsu, CH (2013). Alternative rule induction methods based on incremental objects using rough set theory. *Applied Soft Computing Journal*, 13(1), 372–389. <https://doi.org/10.1016/j.asoc.2012.08.042>
- Inuiguchi, M., & Miyajima, T. (2007). Rough set based rule induction from two decision tables. *European Journal of Operational Research*, 181(3), 1540–1553. <https://doi.org/10.1016/j.ejor.2005.11.054>
- Ishibuchi, H., & Nakashima, T. (2001). Three-objective genetics-based machine learning for linguistic rule extraction. 136.
- Ishibuchi, H., Nakashima, T., & Morisawa, T. (1997). Simple fuzzy rule-based classification systems perform well on commonly used real-world data sets. *Annual Conference of the North American Fuzzy Information Processing Society - NAFIPS*, 251–256. <https://doi.org/10.1109/nafips.1997.624046>
- Ishibuchi, H., Nakashima, T., & Murata, T. (1999). Performance Evaluation of Fuzzy Classifier Systems for Multidimensional Patterns. 29(5), 601–618.
- Ishibuchi, H., & Nojima, Y. (2007). Analysis of interpretability-accuracy tradeoff of fuzzy systems by multiobjective fuzzy genetics-based machine learning. *International Journal of Approximate Reasoning*, 44(1), 4–31. <https://doi.org/10.1016/j.ijar.2006.01.004>
- Ishibuchi, H., Nozaki, K., & Tanaka, H. (1993). Efficient fuzzy partition of pattern space for classification problems. *Fuzzy Sets and Systems*, 59(3), 295–304. [https://doi.org/10.1016/0165-0114\(93\)90474-V](https://doi.org/10.1016/0165-0114(93)90474-V)
- Ishibuchi, H., & Yamamoto, T. (2004). Fuzzy rule selection by multi-objective genetic local search algorithms and rule evaluation measures in data mining. *Fuzzy Sets and Systems*, 141(1), 59–88. [https://doi.org/10.1016/S0165-0114\(03\)00114-3](https://doi.org/10.1016/S0165-0114(03)00114-3)
- Ishibuchi, H., & Yamamoto, T. (2005). Rule weight specification in fuzzy rule-based classification systems. *IEEE Transactions on Fuzzy Systems*, 13(4), 428–435. <https://doi.org/10.1109/TFUZZ.2004.841738>
- Jia, L., Ding, W., & Jiao, H. (2015). Rough-set belief rule model using multinomial subjective logic. *IET Science, Measurement and Technology*, 9(3), 362–366. <https://doi.org/10.1049/iet-smt.2014.0015>
- Jin, Y. (2015). *Advanced Fuzzy Systems Design and Application (Vol. 3)*. Retrieved from <http://repositorio.unan.edu.ni/2986/1/5624.pdf>
- Khoo, L. P., & Zhai, L. Y. (2001). A prototype genetic algorithm-enhanced rough set-based rule induction system. *Computers in Industry*, 46(1), 95–106. [https://doi.org/10.1016/S0166-3615\(01\)00117-8](https://doi.org/10.1016/S0166-3615(01)00117-8)
- Kumar, M., & Yadav, N. (2015). Fuzzy Rough Sets and Its Application in Data Mining Field. 2(3), 237–240.
- Lin, K. P., Hung, K. C., & Lin, C. L. (2018). Rule Generation Based on Novel Kernel Intuitionistic Fuzzy Rough Set Model. *IEEE Access*, 6(c), 11953–11958. <https://doi.org/10.1109/ACCESS.2018.2809456>
- Luo, HC, & Zhong, Y. Bin. (2010). Application and design of MEDS based on rough set data mining rules. *Advances in Intelligent and Soft Computing*, 78, 683–692. https://doi.org/10.1007/978-3-642-14880-4_75
- Ma Tinghuai, Leng Jiazhao, Cui Mengmeng, TW (2009). Inducing Positive and Negative Rules Based on Rough Set. *Information Technology Journal*, 8(7), 1039–1043.
- Magdalena, L. (nd). Fuzzy Rule-Based System. In *Fuzzy Logic* (pp. 203–218).
- Mak, B., & Munakata, T. (2002). Rule extraction from expert heuristics: A comparative study of rough sets with neural networks and ID3. *European Journal of Operational Research*, 136(1), 212–229. [https://doi.org/10.1016/S0377-2217\(01\)00062-5](https://doi.org/10.1016/S0377-2217(01)00062-5)
- Marín-Blázquez, J. G., & Shen, Q. (2002). From approximative to descriptive fuzzy classifiers. *IEEE Transactions on Fuzzy Systems*, 10(4), 484–497. <https://doi.org/10.1109/TFUZZ.2002.800687>
- Meng, Z., & Lu, J. (2016). A Rule-based Service Customization Strategy for Smart Home Context-Aware Automation. *IEEE Transactions on Mobile Computing*, 15(3), 558–571. <https://doi.org/10.1109/TMC.2015.2424427>
- Meng, Zuqiang, & Shi, Z. (2012). Extended rough set-based attribute reduction in inconsistent incomplete

- decision systems. *Information Sciences*, 204, 44–69. <https://doi.org/10.1016/j.ins.2012.04.004>
- Nozaki, K., Tanaka, H., & Ishibuchi, H. (1992). Distributed representation of fuzzy rules and its application to pattern classification. *Fuzzy Sets and Systems*, 52(1), 21–32.
- Othman, ML, Aris, I., Othman, MR, & Osman, H. (2011). Rough-Set-and-Genetic-Algorithm based data mining and Rule Quality Measure to hypothesize distance protective relay operation characteristics from relay event report. *International Journal of Electrical Power and Energy Systems*, 33(8), 1437–1456. <https://doi.org/10.1016/j.ijepes.2011.06.024>
- Pal, S. K., Mitra, S., & Mitra, P. (2003). Rough-fuzzy MLP: Modular evolution, rule generation, and evaluation. *IEEE Transactions on Knowledge and Data Engineering*, 15(1), 14–25. <https://doi.org/10.1109/TKDE.2003.1161579>
- Pattaraintakorn, P., Cercone, N., & Naruedomkul, K. (2006). Rule learning: Ordinal prediction based on rough sets and soft computing. *Applied Mathematics Letters*, 19(12), 1300–1307. <https://doi.org/10.1016/j.aml.2005.08.004>
- Pawlak, Zdzislaw. (2004). Some Issues on Rough Sets. 1–2.
- Pawlak, Zdzislaw. (1982). Rough sets. *International Journal of Computer & Information Sciences*, 11(5), 341–356. <https://doi.org/10.1007/BF01001956>
- Prefecture, O. (1896). Selecting Fuzzy Rules with Forgetting in Fuzzy Classification Systems. 1–6.
- Qian, Y., Liang, J., & Dang, C. (2008). Converse approximation and rule extraction from decision tables in rough set theory. *Computers and Mathematics with Applications*, 55(8), 1754–1765. <https://doi.org/10.1016/j.camwa.2007.08.031>
- Science, I. (2014). Fuzzy If-Then Rules Classifier on Ensemble Data Author Downloaded from Griffith Research Online.
- Shi, F., Sun, S., & Xu, J. (2012). Employing rough sets and association rule mining in KANSEI knowledge extraction. *Information Sciences*, 196, 118–128. <https://doi.org/10.1016/j.ins.2012.02.006>
- Sitompul, Opim Salim; Nababan, Erna Budhiarti; Alim, Z. (2017). International Conference on Information & Communication Technology and Systems (ICTS). Adaptive Distributed Grid- Partition in Generating Fuzzy Rules, 119–124. IEEE.
- Soua, B., Borgi, A., & Tagina, M. (2009). Attributes Regrouping In Fuzzy Rule Based Classification Systems. *International Conference on Signals, Circuits and Systems Attributes*, 1–6.
- Soua, B., Borgi, A., & Tagina, M. (2013). An ensemble method for fuzzy rule-based classification systems. *Knowledge and Information Systems*, 36(2), 385–410. <https://doi.org/10.1007/s10115-012-0532-7>
- Sumalatha, L., Uma Sankar, P., & Sujatha, B. (2016). Rough set based decision rule generation to find behavioral patterns of customers. *Sadhana - Academy Proceedings in Engineering Sciences*, 41(9), 985–991. <https://doi.org/10.1007/s12046-016-0528-1>
- Teoh, HJ, Cheng, CH, Chu, HH, & Chen, JS (2008). Fuzzy time series model based on probabilistic approach and rough set rule induction for empirical research in stock markets. *Data and Knowledge Engineering*, 67(1), 103–117. <https://doi.org/10.1016/j.datak.2008.06.002>
- Vashist, R., & Garg, M. . (2011). Rule Generation based on Reduct and Core: A Rough Set Approach. *International Journal of Computer Applications*, 29(9), 1–5. <https://doi.org/10.5120/3595-4989>
- Wang, X., Yang, J., Jensen, R., & Liu, X. (2006). Rough set feature selection and rule induction for prediction of malignancy degree in brain glioma. *Computer Methods and Programs in Biomedicine*, 83(2), 147–156. <https://doi.org/10.1016/j.cmpb.2006.06.007>
- Yang, X. (2012). Indiscernibility Relations, Rough Sets and Information Systems. In *Incomplete Information Systems and Rough Set Theory*. https://doi.org/10.1007/978-3-642-25935-7_1
- Yu-Neng, Chun-Che Huang, C.-CC (2008). Rule Induction Based on an Incremental Rough Set. *International Joint Conference on Neural Networks*, 5, 1208–1215.