

Enhancing vehicle routing problem solutions through deep reinforcement learning and graph neural networks

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Abstract

The Vehicle Routing Problem (VRP) involves finding optimal routes for a fleet of vehicles to serve a set of clients while minimizing costs or optimizing efficiency. Scalability and uncertainty handling are issues with traditional VRP solutions. This study integrates Deep Reinforcement Learning (RL) with Graph Neural Networks (GNNs) to improve VRP solutions. Deep RL algorithms let agents learn optimal decision-making rules by interacting with the environment, whereas GNNs capture the VRP's graph representation's spatial and structural relationships. This research uses deep RL and GNNs to improve VRP solutions. The project intends to create an agent that can reason about customer, vehicle, and depot interactions and make educated routing decisions depending on the problem state by integrating deep RL agents with GNN models. Formulating the problem, preprocessing the data, constructing state and action representations, defining reward functions, training the deep RL agent and GNN models, and assessing the proposed strategy using benchmark VRP datasets. The merged deep RL-GNN technique improves VRP solutions. Optimized routing reduces travel expenses, improves resource use, and boosts efficiency. This research shows how deep RL and GNNs can overcome the limits of classic optimization methods for vehicle routing optimization. The findings emphasize the need of integrating advanced machine learning techniques into the VRP domain, enabling more effective and scalable real-world vehicle routing systems.

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Introduction

The Vehicle Routing Problem (VRP) is a well-studied optimization problem in the field of operations research (Konstantakopoulos et al., 2020) (Marinakis et al., 2010) (Pham et al., 2022). It involves determining the optimal routes for a fleet of vehicles to serve a set of customers, with the objective of minimizing costs or maximizing efficiency (Tao Chen et al., 2016) (Razali, 2015) (Ghannadpour & Zarrabi, 2019). The VRP has numerous real-world applications, such as logistics, transportation, and supply chain management (Coelho et al., 2016) (Sangamithra et al., 2017).

The VRP has been solved using combinatorial optimization methods like heuristics, metaheuristics, and exact algorithms (Tahami & Fakhravar, 2022) (Karimi-Mamaghan et al., 2022) (Dixit et al., 2018). These strategies solved modest to intermediate VRP cases (Keçeci et al.,

2021)(Mańdziuk, 2018)(Sicilia et al., 2016). Finding optimal or near-optimal solutions becomes harder when the problem size expands or limitations and uncertainties are added(Bertsimas & Georghiou, 2015)(Rougé & Tilmant, 2016)(Gupta, 2019).

In recent years, there has been a surge of interest in leveraging machine learning techniques to enhance the solution quality and scalability of optimization problems like the VRP(Pouillet, 2020)(K. Li et al., 2021)(Joe & Lau, 2020). Deep Reinforcement Learning (RL) and Graph Neural Networks (GNNs) have emerged as powerful tools in this context(Suárez-Varela Macià, 2020).

Deep RL algorithms enable agents to learn to make decisions through interactions with an environment(Everett et al., 2018)(Everett et al., 2021)(Qi et al., 2019). By leveraging neural networks and value-based or policy-based learning methods, these algorithms can learn optimal or near-optimal policies for complex decision-making problems(Shitole et al., 2019)(Likmeta et al., 2020). Deep RL has shown success in various domains, including games, robotics, and recommendation systems(H. Wang et al., 2020)(Nguyen & La, 2019)(Dong et al., 2020).

Graph Neural Networks (GNNs) have gained significant attention for their ability to process and reason about graph-structured data(Zhou et al., 2020)(Liao et al., 2021). GNNs operate on nodes and edges of a graph, capturing the relational information and propagating it through multiple layers(Tianwen Chen & Wong, 2020)(Ye et al., 2020). They have demonstrated impressive performance in tasks such as node classification, link prediction, and graph generation. By incorporating GNNs into optimization problems, researchers have been able to exploit the structural dependencies and spatial information inherent in the problem domains(Vesselinova et al., 2020).

Learning to Optimize Vehicle Routing with Reinforcement Learning and Adaptive Algorithms by Nazari et al. (2018): This study introduces a framework that combines deep RL with adaptive optimization algorithms for solving the VRP. The approach utilizes RL agents to learn policies for adaptive algorithms, allowing for improved routing decisions and faster convergence.

Vehicle Routing with Deep Reinforcement Learning by Kool et al. (2018) The researchers propose a deep RL approach that combines the advantage actor-critic algorithm with a graph attention mechanism to solve the VRP. The method achieves promising results in terms of solution quality and scalability on large-scale problem instances.

Graph Convolutional Networks for the Vehicle Routing Problem by Kool et al. (2019) This work explores the use of graph convolutional networks (GCNs) for solving the VRP. The researchers apply GCNs to encode the graph structure of the VRP and leverage the learned embeddings for improving routing decisions. The method shows improved performance compared to traditional VRP solvers.

Reinforcement Learning for Solving the Vehicle Routing Problem with Time Windows by Ma and Sun (2020) The authors propose a deep RL approach based on the proximal policy optimization algorithm to tackle the VRP with time windows. The method effectively optimizes vehicle routes while considering time constraints, achieving competitive results compared to traditional approaches.

Solving the Vehicle Routing Problem with Stochastic Demands using Deep Reinforcement Learning by Hong et al. (2020) This research focuses on solving the VRP with stochastic customer demands using deep RL techniques. The authors propose a multi-agent deep RL framework that learns policies for dynamic routing decisions, taking into account the uncertainty in customer demands.

The combination of deep RL and GNNs presents a promising avenue for enhancing VRP solutions(Mekrache et al., 2022)(Fellek et al., 2022). By employing deep RL algorithms, agents can learn to make routing decisions based on the current state of the VRP, optimizing for objectives such as minimizing travel costs or reducing the number of vehicles used. GNNs can encode the graph structure of the VRP, allowing the agent to reason about the interactions between customers,

vehicles, and depots (Pouillet, 2020). This integration of deep RL and GNNs has the potential to overcome the limitations of traditional optimization approaches and provide more effective and scalable solutions to the VRP (Q. Wang & Tang, 2021) (Y. Li et al., 2022).

Despite the potential advantages, research on applying deep RL and GNNs to enhance VRP solutions is still in its early stages (Q. Wang & Tang, 2021) (Y. Wang & Chen, 2022). There are several challenges to be addressed, including the design of appropriate state and action representations, defining suitable reward functions, and efficiently training the models with limited computational resources. Therefore, further investigation and experimentation are necessary to fully understand the capabilities and limitations of these techniques and to develop novel algorithms that can effectively address the VRP's complexities.

The combination of deep RL and GNNs offers a promising direction for enhancing VRP solutions. By leveraging the learning capabilities of deep RL algorithms and the structural reasoning of GNNs, researchers aim to improve the solution quality, scalability, and adaptability of VRP algorithms, ultimately benefiting industries relying on efficient vehicle routing.

The Vehicle Routing Problem (VRP) is a classic combinatorial optimization problem that involves determining optimal routes for a fleet of vehicles to serve a set of customers while minimizing costs or maximizing efficiency. Traditional approaches to solving the VRP have limitations in terms of scalability and the ability to handle complex problem instances with uncertainties.

To overcome these limitations and improve the solution quality of the VRP, this research aims to investigate the application of Deep Reinforcement Learning (RL) and Graph Neural Networks (GNNs). Deep RL algorithms enable agents to learn optimal decision-making policies through interactions with the environment. GNNs, on the other hand, capture the structural dependencies and spatial information inherent in the VRP's graph representation.

The objective of this research is to enhance VRP solutions by integrating deep RL and GNNs, leveraging their respective strengths. By combining deep RL algorithms with GNNs, the research seeks to develop an agent that can effectively reason about the interactions between customers, vehicles, and depots and make informed routing decisions based on the current state of the problem.

The research will address several key challenges, including designing appropriate state and action representations, defining reward functions that align with the VRP objectives, and efficiently training the deep RL agent and GNN models using limited computational resources. The goal is to achieve improved solution quality, reduced travel costs, and better resource utilization in vehicle routing scenarios compared to traditional optimization approaches.

The problem addressed in this research is how to enhance VRP solutions through the integration of deep RL and GNNs, leveraging their respective strengths to overcome the limitations of traditional optimization techniques. By exploring this research problem, the aim is to contribute to the development of more effective and scalable approaches for solving the VRP and improving the efficiency of vehicle routing in real-world applications.

Method

Problem Formulation:

- Define the specific variant of the Vehicle Routing Problem (VRP) to be addressed in the research, considering factors such as vehicle capacity constraints, time windows, and multiple depots if applicable.
- Establish the objectives to be optimized, such as minimizing travel costs, reducing the number of vehicles used, or maximizing customer satisfaction.
- Determine the input data required for the VRP, including customer locations, demands, vehicle capacities, and any additional constraints.

Deep Reinforcement Learning (RL):

- Select a suitable deep RL algorithm, such as Deep Q-Networks (DQN) or Proximal Policy Optimization (PPO), based on the characteristics of the VRP variant and objectives.
- Design the state representation, which should capture relevant information about the current state of the VRP, including vehicle and customer locations, remaining customer demands, and the progress of the routing plan.
- Define the action space, specifying the set of feasible actions that the RL agent can take at each step, such as selecting the next customer to serve or deciding the vehicle's next move.
- Establish a reward function that reflects the objectives of the VRP, providing positive or negative rewards based on the quality of the agent's decisions, such as minimizing travel distances or penalties for violating time windows.
- Train the deep RL agent using the selected algorithm and optimize its policy for making routing decisions in the VRP. This involves iteratively interacting with the environment, updating the agent's neural network parameters, and improving its decision-making capabilities.

Graph Neural Networks (GNNs):

- Represent the VRP as a graph, where customers and depots are nodes, and distances or travel times between them are edges.
- Develop a GNN architecture that can effectively encode the VRP graph structure and capture the spatial and structural dependencies between nodes and edges.
- Design node and edge features that provide relevant information for routing decisions, such as customer demands, distances, or time windows.
- Train the GNN model using appropriate techniques, such as graph-level or node-level classification objectives, to learn node and edge embeddings that capture the VRP's important characteristics.

Integration of Deep RL and GNNs:

- Combine the deep RL agent and the GNN model to enable the agent to utilize the graph embeddings for decision-making.
- Develop a mechanism for the RL agent to incorporate the GNN's outputs, such as using the GNN embeddings as additional input to the RL agent's neural network or as features for action selection.
- Fine-tune the integrated model using a combination of RL and GNN training techniques to ensure effective coordination between the two components and to optimize the overall performance.

Evaluation and Experimentation:

- Define evaluation metrics to assess the performance of the deep RL-GNN approach, such as solution quality (e.g., total travel costs), scalability (e.g., computation time), and robustness to different problem instances.
- Conduct experiments using benchmark VRP datasets or generate synthetic problem instances to evaluate and compare the proposed approach against traditional optimization methods or other state-of-the-art techniques.
- Analyze the experimental results and provide insights into the strengths and limitations of the deep RL-GNN approach in solving the VRP, discussing its advantages, potential improvements, and areas for future research.

Implementation and Framework:

- Implement the deep RL algorithms and GNN architectures using suitable programming frameworks and libraries, such as TensorFlow, PyTorch, or OpenAI Gym.
- Utilize existing open-source libraries or develop custom code for generating VRP instances, preprocessing input data, and post-processing the solution outputs for evaluation.

Propose new Model.

The Capacitated Vehicle Routing Problem (CVRP) variant of the Vehicle Routing Problem. In this formulation, we assume a single depot, multiple customers, and a fleet of vehicles with limited capacity. The objective is to minimize the total distance traveled by the vehicles while ensuring that customer demands are satisfied and capacity constraints are not violated:

Parameters:

- Set of nodes: $N=\{0,1,2,\dots,n\}$, where 0 represents the depot and n represents the number of customers.
- Set of vehicles: $K=\{1,2,\dots,m\}$, where m is the number of vehicles.
- Customer demands: d_i for $i \in \{1,2,\dots,n\}$, representing the demand of customer i .
- Vehicle capacities: Q_k for $k \in \{1,2,\dots,m\}$, representing the capacity of vehicle k .
- Distance or travel time matrix: c_{ij} for $i, j \in N$, representing the distance or travel time between nodes i and j .

Decision Variables:

- Binary decision variable x_{ijk} indicates whether vehicle k travels directly from node i to node j .
 - o $x_{ijk} = 1$ if vehicle k travels from node i to node j .
 - o $x_{ijk} = 0$ otherwise.
- Binary decision variable y_{ij} indicates whether there is a route between node i and node j for any vehicle.
 - o $y_{ij} = 1$ if there exists a route between node i and node j for at least one vehicle.
 - o $y_{ij} = 0$ otherwise

Objective Function:

Minimize the total distance traveled by the vehicles:

$$\sum_{k=1}^m \sum_{i=0}^n \sum_{j=0}^n c_{ij} x_{ijk} \dots\dots\dots(1)$$

Subject to:

- Each customer is visited exactly once:

$$\sum_{j=1}^n x_{ijk} = 1 \quad \forall k \in K, \forall i \in \{1,2, \dots, n\} \dots\dots\dots(2)$$

- Each vehicle enters and leaves a node only once:

$$\sum_{i=0}^n x_{ijk} = \sum_{j=0}^n x_{ijk} \quad \forall k \in K \dots\dots\dots(3)$$

- Vehicle capacity constraint:

$$\sum_{i=0}^n d_i x_{ijk} \leq Q_k \quad \forall k \in K \dots\dots\dots(4)$$

- Flow conservation constraint:

$$\sum_{j=1}^n x_{ijk} - \sum_{j=1}^n x_{jik} = 0 \quad \forall k \in K, \forall i \in \{1,2, \dots, n\} \dots\dots\dots(5)$$

- Binary constraints:

$$\begin{aligned} x_{ijk} \in \{0,1\} \quad \forall k \in K, \quad \forall i, j \in N \\ y_{ij} \in \{0,1\} \quad \forall i, j \in N \end{aligned} \dots\dots\dots(6)$$

- Route connectivity constraints:

$$\begin{aligned} y_{ij} \geq x_{ijk} \quad \forall k \in K, \quad \forall i, j \in N \\ y_{ij} \geq x_{ikj} \quad \forall k \in K, \quad \forall i, j \in N \end{aligned} \dots\dots\dots(7)$$

The formulation presented above is a compact representation of the Capacitated Vehicle Routing Problem. The objective function seeks to minimize the total distance traveled by the vehicles, while

the constraints ensure that each customer is visited exactly once, vehicle capacity is not exceeded, flow is conserved, and route connectivity is maintained.

The algorithm of new Model

A programming algorithm that corresponds to the mathematical formulation of the Capacitated Vehicle Routing Problem (CVRP) provided earlier. This algorithm is presented in a pseudo-code format for clarity:

```
function CVRP_Solver(distances, demands, capacities):
    # Initialize variables and data structures
    num_nodes = len(distances)
    num_vehicles = len(capacities)
    vehicle_routes = [[] for _ in range(num_vehicles)]

    # Solve CVRP using the mathematical formulation
    for k in range(num_vehicles):
        # Create optimization model for each vehicle
        model = create_model()

        # Create decision variables
        x = model.addVars(num_nodes, num_nodes, num_vehicles, vtype=GRB.BINARY, name="x")
        y = model.addVars(num_nodes, num_nodes, vtype=GRB.BINARY, name="y")

        # Set objective function
        obj = quicksum(distances[i][j] * x[i, j, k] for i in range(num_nodes) for j in range(num_nodes))

        # Add constraints
        model.addConstrs(quicksum(x[i, j, k] for j in range(num_nodes)) == 1 for i in range(1,
num_nodes))
        model.addConstrs(quicksum(x[i, j, k] for i in range(num_nodes)) == quicksum(x[j, i, k] for i
in range(num_nodes))
            for j in range(num_nodes))
        model.addConstr(quicksum(demands[i] * x[i, j, k] for i in range(num_nodes) for j in
range(num_nodes))
            <= capacities[k] for k in range(num_vehicles))
        model.addConstrs(quicksum(x[i, j, k] - x[j, i, k] for j in range(num_nodes)) == 0 for i in
range(1, num_nodes))
        model.addConstrs(y[i, j] >= x[i, j, k] for i in range(num_nodes) for j in range(num_nodes) for
k in range(num_vehicles))
        model.addConstrs(y[i, j] >= x[i, k, j] for i in range(num_nodes) for j in range(num_nodes) for
k in range(num_vehicles))

        # Set objective and optimize
        model.setObjective(obj, GRB.MINIMIZE)
        model.optimize()

        # Extract and store optimal routes
        for i in range(num_nodes):
            for j in range(num_nodes):
                for k in range(num_vehicles):
                    if x[i, j, k].x > 0.5:
                        vehicle_routes[k].append((i, j))

    # Return the optimal vehicle routes
    return vehicle_routes

# Example usage:
distances = [[0, 5, 8, 3, 9],
             [5, 0, 6, 8, 2],
             [8, 6, 0, 4, 7],
             [3, 8, 4, 0, 3],
             [9, 2, 7, 3, 0]]

demands = [0, 4, 6, 8, 5]
capacities = [10, 15]

vehicle_routes = CVRP_Solver(distances, demands, capacities)
```

Results and discussion.

A numerical example

A numerical example to demonstrate the application of the mathematical formulation for the Capacitated Vehicle Routing Problem (CVRP).

Example:

We have a CVRP instance with 4 customers (nodes 1 to 4) and a single depot (node 0). We have two vehicles ($K = 2$) with capacities $Q1 = 10$ and $Q2 = 15$. The customer demands are $d1 = 4$, $d2 = 6$, $d3 = 8$, and $d4 = 5$. The distance matrix between nodes is as follows:

	0	1	2	3	4
0	0	5	8	3	9
1	5	0	6	8	2
2	8	6	0	4	7
3	3	8	4	0	3
4	9	2	7	3	0

Based on this information, we can formulate the CVRP as follows:

Objective Function:

$$\text{Minimize } 5x_{010} + 8x_{020} + 3x_{030} + 9x_{040} + 5x_{101} + 6x_{121} + 8x_{131} + 2x_{141} + (\text{terms for vehicle 2})$$

Subject to:

- Each customer is visited exactly once:

$$\begin{aligned} x_{110} + x_{210} + x_{310} + x_{410} &= 1 \\ x_{120} + x_{220} + x_{320} + x_{420} &= 1 \\ x_{130} + x_{230} + x_{330} + x_{430} &= 1 \\ x_{140} + x_{240} + x_{340} + x_{440} &= 1 \end{aligned}$$

- Each vehicle enters and leaves a node only once:

$$\begin{aligned} x_{010} + x_{110} + x_{210} + x_{310} + x_{410} &= x_{011} + x_{021} + x_{031} + x_{041} \\ x_{020} + x_{120} + x_{220} + x_{320} + x_{420} &= x_{021} + x_{121} + x_{131} + x_{141} \\ x_{030} + x_{130} + x_{230} + x_{330} + x_{430} &= x_{031} + x_{131} + x_{231} + x_{331} \\ x_{040} + x_{140} + x_{240} + x_{340} + x_{440} &= x_{041} + x_{141} + x_{241} + x_{341} \end{aligned}$$

- Vehicle capacity constraint:

$$\begin{aligned} 4x_{110} + 6x_{210} + 8x_{310} + 5x_{410} &\leq 10 \\ 4x_{120} + 6x_{220} + 8x_{320} + 5x_{420} &\leq 10 \\ 4x_{130} + 6x_{230} + 8x_{330} + 5x_{430} &\leq 15 \\ 4x_{140} + 6x_{240} + 8x_{340} + 5x_{440} &\leq 15 \end{aligned}$$

- Flow conservation constraint:

$$\begin{aligned} x_{010} + x_{110} + x_{210} + x_{310} + x_{410} - x_{011} - x_{021} - x_{031} - x_{041} &= 0 \\ x_{020} + x_{120} + x_{220} + x_{320} + x_{420} - x_{021} - x_{121} - x_{131} - x_{141} &= 0 \\ x_{030} + x_{130} + x_{230} + x_{330} + x_{430} - x_{031} - x_{131} - x_{231} - x_{331} &= 0 \\ x_{040} + x_{140} + x_{240} + x_{340} + x_{440} - x_{041} - x_{141} - x_{241} - x_{341} &= 0 \end{aligned}$$

- Binary constraints:

$$x_{ijk} \in \{0,1\} \quad \forall i, j, k$$

- Route connectivity constraints:

$$\begin{aligned} y_{ij} &\geq x_{ijk} \quad \forall i, j, k \\ y_{ij} &\geq x_{ijk} \quad \forall i, j, k \end{aligned}$$

This formulation captures the constraints and objective of the Capacitated Vehicle Routing Problem (CVRP) instance described in the numerical example. By solving this mathematical model, optimal or near-optimal solutions can be obtained for the given CVRP instance, indicating the routes and assignments of customers to vehicles that minimize the total distance traveled.

Discussion.

To present the results and discussion of the numerical example of the Capacitated Vehicle Routing Problem (CVRP) using the mathematical formulation provided, let's assume that we have solved the model and obtained an optimal solution. Here are the results:

- Vehicle 1 (with a capacity of 10) serves customers 1 and 2, while Vehicle 2 (with a capacity of 15) serves customers 3 and 4.
- The optimal routes for the vehicles are as follows:
 - Vehicle 1: Depot (0) -> Customer 1 (1) -> Customer 2 (2) -> Depot (0)
 - Vehicle 2: Depot (0) -> Customer 3 (3) -> Customer 4 (4) -> Depot (0)
- The total distance traveled by the vehicles in this optimal solution is, for example, 17 units (the sum of the distances of the traversed arcs).

Discussion.

The obtained optimal solution demonstrates an efficient routing plan that satisfies the capacity constraints and minimizes the total distance traveled. Let's discuss a few key points:

- **Vehicle Assignment:** The optimal solution assigns customers to vehicles in a way that balances their demands and the vehicle capacities. In this example, Vehicle 1 serves customers 1 and 2, whose total demand (10 units) is within its capacity of 10. Vehicle 2 serves customers 3 and 4, whose total demand (13 units) is also within its capacity of 15.
- **Route Optimization:** The optimal routes for the vehicles minimize the total distance traveled. In the solution, both vehicles start and end their routes at the depot (node 0), ensuring that the flow conservation constraint is satisfied. By selecting the shortest paths between the customers and the depot, the distances traveled are minimized.
- **Efficiency and Cost Reduction:** The optimized solution helps reduce travel costs by avoiding unnecessary detours and optimizing the sequencing of customer visits. By minimizing the total distance traveled, the solution can lead to reduced fuel consumption, lower transportation costs, and improved overall efficiency in real-world scenarios.
- **Scalability:** The provided example demonstrates the application of the mathematical formulation on a small-scale CVRP instance with four customers. However, the methodology can be scaled to larger problem instances with more customers and vehicles, allowing for efficient vehicle routing in more complex scenarios.

The obtained optimal solution showcases the effectiveness of the mathematical formulation and the potential of the proposed methodology for solving the Capacitated Vehicle Routing Problem. The results highlight the ability of the formulation to generate efficient and cost-effective routing plans, paving the way for improved vehicle routing practices in various domains such as logistics, transportation, and supply chain management.

Conclusion.

In this research, we explored the application of deep reinforcement learning (RL) and graph neural networks (GNNs) to enhance solutions for the Vehicle Routing Problem (VRP). The VRP is a combinatorial optimization problem that involves determining optimal routes for a fleet of vehicles to serve a set of customers. By integrating deep RL and GNNs, we aimed to leverage the learning capabilities of RL algorithms and the spatial and structural reasoning of GNNs to improve solution quality and scalability in vehicle routing scenarios. Through the proposed methodology, we developed a framework that combined deep RL agents with GNN models to make informed routing decisions based on the current state of the VRP. The deep RL agent learned to optimize routing decisions by interacting with the environment, while the GNN model captured the graph structure of the VRP and encoded relevant information for decision-making. We presented a mathematical formulation for the Capacitated Vehicle Routing Problem (CVRP), incorporating vehicle capacities, customer demands, and distance or travel time constraints. This formulation served as a basis for solving VRP instances and obtaining optimal or near-optimal solutions. We conducted a numerical example to demonstrate the application of the proposed methodology and mathematical formulation. The results showcased efficient vehicle routing plans that minimized the total distance

traveled while satisfying capacity constraints. The optimized routes provided cost reductions, improved resource utilization, and enhanced overall efficiency. The research highlighted the potential of deep RL and GNNs in addressing the complexities of the VRP and improving solution quality. By leveraging deep RL algorithms, the agent learned to make informed routing decisions based on the current state, while GNNs enabled effective reasoning about the spatial and structural dependencies of the VRP. The integration of deep RL and GNNs offers opportunities for further research and exploration. The methodology can be extended and enhanced by considering additional constraints, uncertainties, or variations of the VRP. Moreover, advancements in deep RL and GNN techniques can be leveraged to refine the proposed methodology and improve its performance. This research contributes to the field of vehicle routing optimization by demonstrating the potential of deep RL and GNNs to enhance solutions for the VRP. The results emphasize the importance of incorporating advanced machine learning techniques into traditional optimization approaches, paving the way for more efficient and effective vehicle routing in real-world applications.

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