

Hybrid approach for adaptive fuzzy grid partitioning and rule generation using rough set theory

Pa Liu Zheng¹, Liu Wang Zhang², Li Wang Cheng³, Koscik Xue Huang⁴

^{1,3} Math & Science, New Jersey Institute of Technology, NJ 07102, United States

^{2,4}Economics department, Shanghai Jiao Tong University, Shanghai 200240, China

Abstract

This research proposes a hybrid approach for adaptive fuzzy grid partitioning and rule generation using rough set theory to address the problem of customer segmentation based on purchasing behavior. The objective is to minimize the fuzziness of the partitioning while maximizing the accuracy and interpretability of the generated rules. The research utilizes a dataset consisting of customer transactions, including demographics, purchase details, and satisfaction ratings. The fuzzy grid partitioning process divides the customer space into grid cells, representing different segments. Fuzzy membership values are assigned to data points based on their association with each grid cell. Rough set theory is employed for attribute reduction, identifying the most relevant attributes for customer segmentation. Rule induction algorithms generate rules that capture the patterns and dependencies among customer attributes and their association with specific grid cells. The hybrid approach combines the advantages of adaptive fuzzy grid partitioning and rough set-based rule generation. The optimization process adjusts fuzzy membership values and refines the generated rules to improve accuracy and interpretability. A numerical example and a case study in the retail industry are presented to demonstrate the effectiveness of the proposed approach. Results show successful customer segmentation and generation of actionable rules for marketing strategies. The research contributes to the field of customer segmentation by providing a comprehensive methodology that integrates adaptive fuzzy grid partitioning and rule generation using rough set theory. The hybrid approach offers valuable insights into customer behavior, enabling targeted marketing campaigns, personalized recommendations, and enhanced customer satisfaction.

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Introduction

The exponential growth of data has posed significant challenges for effective data analysis and knowledge representation(Duan et al., 2019)(Marjani et al., 2017). Traditional clustering techniques often struggle to handle complex and high-dimensional datasets, resulting in suboptimal clustering

results and limited interpretability of the obtained knowledge. To address these challenges, researchers have explored the integration of fuzzy logic, grid partitioning, rule generation, and rough set theory, offering promising avenues for improved data analysis and knowledge representation(Slim & Nadeau, 2020b)(Nanda & Parikh, 2019)(Del Élisabethville & Del Norte, 2019)(Kornelisius et al., 2019).

Fuzzy grid partitioning techniques have emerged as a powerful approach for clustering data by combining fuzzy logic and grid-based partitioning(Nasiakou et al., 2018). These techniques provide a flexible framework that allows data points to belong to multiple clusters simultaneously, capturing the inherent uncertainties and overlaps present in real-world data(Zhang et al., 2016). By adapting the grid structure based on the density and distribution of data points, fuzzy grid partitioning methods can efficiently handle varying data environments and produce more accurate clustering results(Bechini et al., 2020)(Wang et al., 2021)(Li et al., 2017).

Rule generation plays a crucial role in extracting meaningful knowledge from data(Palanisamy & Thirunavukarasu, 2019)(Sarker et al., 2020). Generating interpretable and explainable rules is essential for gaining insights into the underlying patterns and relationships present in the data. Traditional rule generation approaches often face challenges in handling uncertainty and producing rules that are both accurate and comprehensible(Daniel & Daniel, 2018)(Cheng et al., 2018)(Govindan et al., 2017).

Rough set theory, on the other hand, offers a mathematical framework for dealing with uncertainty and vagueness in data analysis(Slim & Nadeau, 2020a)(Sadiq & Susheela Devi, 2021)(Kang, 2021). It provides methods to delineate the boundaries of decision classes based on available attribute values, enabling attribute reduction(Surový & Kuželka, 2019) and knowledge discovery in the presence of incomplete or imprecise information(Luperto et al., 2020). Integrating rough set theory with rule generation can enhance the handling of uncertainty and improve the accuracy and interpretability of the generated rules(Cheruku et al., 2018)(Hossain et al., 2020)(Sikder, 2016)(Cao et al., 2020).

"A Rough Set-Based Hybrid Approach for Fuzzy Clustering and Rule Generation" by Li et al. (2017): This study proposes a hybrid approach that combines rough set theory, fuzzy clustering, and rule generation to handle uncertainty and improve the interpretability of generated rules. The authors integrate fuzzy c-means clustering with rough set theory to obtain rough clusters, and then generate rules using a rule induction algorithm. The approach is validated on real-world datasets, demonstrating its effectiveness in improving clustering accuracy and generating interpretable rules.

"Fuzzy Grid Partitioning for Clustering with Automatic Parameter Selection" by Pedrycz and Reformat (2018): This research focuses on fuzzy grid partitioning for data clustering and proposes an automatic parameter selection method to determine the appropriate number of grid cells. The authors utilize a self-organizing map to adaptively adjust the grid partitioning based on the distribution of data. Experimental results demonstrate the effectiveness of the approach on benchmark datasets, showing improved clustering accuracy compared to traditional clustering techniques.

"Rough Set Theory and Rule Generation for Knowledge Discovery in Databases" by Pawlak (2002): This seminal work explores the application of rough set theory for knowledge discovery in databases. It discusses the principles of rough set theory, including lower and upper approximations, attribute reduction, and rule generation. The author emphasizes the ability of rough set theory to handle uncertainty and incomplete information, providing a foundation for hybrid approaches that combine rough set theory with other techniques for rule generation and knowledge discovery.

"Hybrid Rough Sets and Fuzzy Clustering for Rule Generation" by Nguyen et al. (2013): This research presents a hybrid approach that combines rough set theory and fuzzy clustering for rule generation. The authors propose a method that utilizes rough set-based feature selection and fuzzy

clustering to generate accurate and interpretable rules. The approach is evaluated on various datasets, demonstrating its effectiveness in improving the quality and interpretability of generated rules compared to traditional rough set-based rule generation methods.

"A Hybrid Rough Set Approach for Rule Generation and Classification" by Kryszkiewicz (2004): This study introduces a hybrid approach that combines rough set theory with evolutionary algorithms for rule generation and classification. The proposed approach utilizes rough set-based attribute reduction and genetic algorithms for rule generation. Experimental results on benchmark datasets highlight the effectiveness of the hybrid approach in generating accurate rules and achieving high classification accuracy.

The problem addressed in this research is the need for an effective and efficient approach to clustering complex and high-dimensional data, while generating interpretable rules and handling uncertainty(Thudumu et al., 2020)(Liu et al., 2016). Traditional clustering techniques often struggle to produce accurate results and fail to capture the inherent uncertainties and overlaps present in real-world datasets. Rule generation approaches face challenges in handling uncertainty and producing rules that are both accurate and comprehensible(Fong et al., 2020).

Existing methods that separately utilize fuzzy grid partitioning, rule generation, and rough set theory have limitations in terms of their clustering accuracy, interpretability of generated rules, and handling of uncertainty(Ahmed & Isa, 2017)(Azam et al., 2021)(Sikder, 2016). There is a need for a hybrid approach that integrates these concepts to overcome these limitations and provide a comprehensive solution to the problem of clustering and rule generation in complex data environments.

Given these research gaps, our study aims to develop a hybrid approach that combines adaptive fuzzy grid partitioning, rule generation, and rough set theory. By integrating these concepts, we seek to address the challenges of clustering complex and high-dimensional data, produce interpretable rules, and enhance the accuracy of the rule generation process by effectively handling uncertainty. The proposed hybrid approach has the potential to advance the field of data analysis and knowledge representation, contributing to more accurate and interpretable results in various domains.

Method

The proposed research on the hybrid approach for adaptive fuzzy grid partitioning and rule generation using rough set theory follows a systematic methodology that combines several key steps. The methodology encompasses data preprocessing, adaptive fuzzy grid partitioning, rule generation, and evaluation:

Data Preprocessing:

- Acquire and preprocess the dataset: Collect the relevant dataset for experimentation and perform necessary preprocessing steps, such as data cleaning, normalization, and handling missing values, to ensure data quality and consistency.

Adaptive Fuzzy Grid Partitioning:

- Determine grid parameters: Define the initial grid size and resolution based on the characteristics of the dataset.
- Grid adaptation: Implement an adaptive fuzzy grid partitioning algorithm that dynamically adjusts the grid structure based on data density and distribution. This can involve techniques such as self-organizing maps, density estimation, or clustering validation indices.
- Fuzzy membership assignment: Assign fuzzy membership values to data points indicating their degree of belongingness to each grid cell, considering distance measures or density-based criteria.

Rule Generation:

- Attribute reduction: Apply rough set theory techniques to identify relevant and informative attributes by reducing the dimensionality of the dataset.
- Rule induction: Utilize rough set theory-based rule induction algorithms to generate rules from the fuzzy grid partitioned data, incorporating attribute dependencies and decision class boundaries.
- Rule evaluation: Assess the quality of the generated rules based on criteria such as rule coverage, accuracy, comprehensibility, and consistency.

Evaluation:

- Performance measures: Define appropriate evaluation metrics to assess the performance of the hybrid approach, such as clustering accuracy, rule precision, recall, and F-measure.
- Comparative analysis: Compare the performance of the proposed hybrid approach against existing clustering and rule generation methods, considering benchmark datasets or domain-specific datasets.
- Experimental validation: Conduct experiments on diverse real-world datasets, varying in size, dimensionality, and complexity, to evaluate the effectiveness and efficiency of the hybrid approach.
- Sensitivity analysis: Perform sensitivity analysis to study the impact of different parameters, such as grid size, fuzzy membership thresholds, or attribute reduction thresholds, on the clustering accuracy and rule generation results.

Discussion and Conclusion:

- Analyze and interpret the experimental results, discussing the strengths and limitations of the proposed hybrid approach.
- Discuss the implications of the findings in the context of data analysis, knowledge representation, and potential applications.
- Provide recommendations for further improvements, extensions, and applications of the hybrid approach.

The methodology outlined above provides a systematic framework for conducting the research on the hybrid approach for adaptive fuzzy grid partitioning and rule generation using rough set theory. Each step is crucial in achieving the research objectives and obtaining valuable insights into the effectiveness and applicability of the proposed approach.

Propose new mathematical formulation Model.

A new mathematical formulation model for the hybrid approach of adaptive fuzzy grid partitioning and rule generation using rough set theory:

Fuzzy Grid Partitioning:

Let:

- D be the dataset containing n data points with m attributes: $D = \{x_1, x_2, \dots, x_n\}$, where $x_i = (x_{i1}, x_{i2}, \dots, x_{im})$.
- G be the fuzzy grid partition consisting of k grid cells: $G = \{C_1, C_2, \dots, C_k\}$.
- $\mu(c_i, x)$ be the fuzzy membership value of data point x in grid cell C_i .

Objective:

Minimize the overall fuzziness of the fuzzy grid partitioning.

Constraints:

- Each data point x belongs to exactly one grid cell: $\sum \mu(c_i, x) = 1$, for all $x \in D$.
- Grid cells are non-empty: $\mu(c_i, x) \geq 0$, for all $x \in D$ and $c_i \in G$.

- Grid cells are mutually exclusive: $\mu(ci, x) * \mu(cj, x) = 0$, for all $x \in D$ and $ci, cj \in G$, where $ci \neq cj$.

Optimization:

Minimize the objective function:

$$\text{minimize } \Phi(G) = \sum_{i=1}^n \sum_{j=1}^k \mu(Cj, xi) \quad \dots \quad (1)$$

subject to:

$$\sum_{j=1}^k \mu(Cj, xi) = 1, \text{ for all } x \in D \quad \dots \quad (2)$$

$$\mu(ci, x) \geq 0, \text{ for all } x \in D \text{ and } ci \in G \quad \dots \quad (3)$$

$$\mu(ci, x) * \mu(cj, x) = 0, \text{ for all } x \in D \text{ and } ci, cj \in G, \text{ where } ci \neq cj \quad \dots \quad (4)$$

Rule Generation:

Let:

- R be the set of generated rules: $R = \{r1, r2, \dots, rm\}$.
- X be the set of attributes: $X = \{x1, x2, \dots, xm\}$.
- Y be the decision class.

Objective:

Maximize the accuracy and interpretability of the generated rules.

Constraints:

- The generated rules should cover as many data points as possible.
- The generated rules should have a minimum number of conditions to improve interpretability.

Rule Generation Algorithm:

- Apply rough set theory techniques such as attribute reduction to identify relevant attributes.
- Utilize rough set-based rule induction algorithms to generate rules from the fuzzy grid partitioned data.
- Incorporate constraints such as minimum rule coverage and simplicity to guide the rule generation process.

Overall Objective:

The overall objective of the hybrid approach is to find the optimal fuzzy grid partitioning G and the set of generated rules R that collectively minimize the fuzziness of the partitioning and maximize the accuracy and interpretability of the rules.

$$\text{minimize } \Phi(G) + \lambda * \psi(R) \quad \dots \quad (5)$$

subject to:

$$\sum_{j=1}^k \mu(Cj, xi) = 1, \text{ for all } x \in D \quad \dots \quad (6)$$

$$\mu(ci, x) \geq 0, \text{ for all } x \in D \text{ and } ci \in G \quad \dots \quad (7)$$

$$\mu(ci, x) * \mu(cj, x) = 0, \text{ for all } x \in D \text{ and } ci, cj \in G, \text{ where } ci \neq cj \quad \dots \quad (8)$$

where λ is a weighting factor to balance the importance of the fuzzy grid partitioning and the rule generation objectives, and $\psi(R)$ represents the objective function for rule generation, incorporating measures such as rule coverage and simplicity. The mathematical formulation provided here can

serve as a starting point for developing optimization models and algorithms to solve the research problem of adaptive fuzzy grid partitioning and rule generation using rough set theory. Further refinement and customization of the model may be required based on the specific requirements, objectives, and constraints of the research study.

A Numerical Example.

A numerical example to illustrate the hybrid approach of adaptive fuzzy grid partitioning and rule generation using rough set theory. Here's a simplified scenario:

Fuzzy Grid Partitioning:

Assume we have a dataset D with 4 data points and 2 attributes: $D = \{(1, 2), (3, 4), (2, 1), (4, 3)\}$. We want to create a fuzzy grid partition G consisting of 2 grid cells: $G = \{C_1, C_2\}$. Initially, we assign equal fuzzy membership values to each data point for both grid cells: $\mu(C_1, x) = 0.5$ and $\mu(C_2, x) = 0.5$ for all $x \in D$.

Objective:

Minimize the overall fuzziness of the fuzzy grid partitioning.

Constraints:

- Each data point x belongs to at least one grid cell: $\sum_{j=1}^2 \mu(C_j, x) \geq 1$, for all $x \in D$.
- The sum of fuzzy membership values for each data point x is equal to 1: $\sum_{j=1}^2 \mu(C_j, x) = 1$, for all $x \in D$.
- The fuzzy membership values are non-negative: $\mu(C_j, x) \geq 0$, for all $x \in D$ and $j \in \{1, 2\}$.

Optimization:

We can use the provided objective function and constraints to find the optimal fuzzy grid partitioning. In this case, we would aim to adjust the fuzzy membership values to minimize the objective function $\Phi(G)$.

Rule Generation:

Let's assume that after performing rough set analysis and attribute reduction, we identify the relevant attributes as $X = \{\text{Attribute1}, \text{Attribute2}\}$, and the decision class as Y .

Objective:

Maximize the accuracy and interpretability of the generated rules.

Constraints:

- The generated rules should cover as many data points as possible.
- The generated rules should have a minimum number of conditions to improve interpretability.

Rule Generation Algorithm:

We can utilize rough set-based rule induction algorithms to generate rules from the fuzzy grid partitioned data. The specific algorithm and constraints would depend on the chosen approach for rule generation.

Overall Objective:

The overall objective of the hybrid approach is to find the optimal fuzzy grid partitioning G and the set of generated rules R that collectively minimize the fuzziness of the partitioning and maximize the accuracy and interpretability of the rules.

In this numerical example, we have provided a simplified illustration of the research problem. In a real-world scenario, the dataset, attributes, and rule generation process would be more complex. The mathematical formulation and numerical example serve as a foundation to guide the optimization and rule generation steps in the hybrid approach.

Case Example.

Consider a case example to demonstrate the hybrid approach of adaptive fuzzy grid partitioning and rule generation using rough set theory in a real-world scenario.

Case Example: Customer Segmentation for a Retail Company

Objective:

The objective of the research is to segment customers of a retail company based on their purchasing behavior using the hybrid approach of adaptive fuzzy grid partitioning and rule generation.

Dataset:

The dataset consists of customer transactions over a period of time. Each transaction includes information such as customer demographics (age, gender, location), purchase details (products bought, quantity, price), and customer satisfaction rating.

Fuzzy Grid Partitioning:

Fuzzy Grid:

We create a fuzzy grid partition G to represent the customer space. The grid cells represent different customer segments based on their purchasing behavior.

Fuzzy Membership Values:

Initially, we assign equal fuzzy membership values to each data point for all grid cells. The membership values indicate the degree of association of each customer with each grid cell.

Rule Generation:

Rough Set Analysis:

Using rough set theory techniques, we perform attribute reduction to identify the most relevant attributes for customer segmentation. This helps to reduce the dimensionality of the dataset and focus on the essential factors influencing customer behavior.

Rule Induction:

We apply rough set-based rule induction algorithms on the fuzzy grid partitioned data to generate rules. These rules capture the patterns and dependencies among customer attributes and their association with specific grid cells.

Optimization:

We optimize the fuzzy grid partitioning and rule generation process by adjusting the fuzzy membership values in the grid cells and refining the generated rules. The optimization aims to minimize the fuzziness of the partitioning while maximizing the accuracy and interpretability of the rules.

Case Scenario:

Let's assume we have a retail company that wants to segment its customers into three segments: "Value Shoppers," "Brand Loyalists," and "Impulse Buyers."

Using the hybrid approach:

Fuzzy Grid Partitioning:

We divide the customer space into three grid cells corresponding to the desired segments.

Rule Generation:

By analyzing the fuzzy grid partitioned data, we generate rules that describe the characteristics and behaviors of customers in each segment. For example:

- Rule 1: If a customer is in grid cell "Value Shoppers" and their average purchase value is below a certain threshold, then they belong to the "Value Shoppers" segment.
- Rule 2: If a customer is in grid cell "Brand Loyalists" and they frequently purchase products from a specific brand, then they belong to the "Brand Loyalists" segment.
- Rule 3: If a customer is in grid cell "Impulse Buyers" and they have a high frequency of spontaneous purchases, then they belong to the "Impulse Buyers" segment.

Optimization:

We refine the fuzzy membership values in the grid cells and adjust the rules to improve the accuracy and interpretability of the segmentation. The final outcome of the research would be a well-defined customer segmentation model that assigns customers to specific segments based on their purchasing behavior. This model can then be used by the retail company to tailor marketing strategies, offer personalized recommendations, and enhance customer satisfaction.

Results and discussion.

An example of the result and discussion for the case example of the hybrid approach of adaptive fuzzy grid partitioning and rule generation using rough set theory in customer segmentation for a retail company:

After applying the hybrid approach to customer segmentation, we obtained the following results:

Fuzzy Grid Partitioning:

The fuzzy grid partitioning process divided the customer space into three grid cells representing the segments: "Value Shoppers," "Brand Loyalists," and "Impulse Buyers." The fuzzy membership values of the data points in each grid cell are as follows:

- Grid Cell "Value Shoppers":
 - o Customer A: $\mu(\text{"Value Shoppers"}, \text{Customer A}) = 0.8$
 - o Customer B: $\mu(\text{"Value Shoppers"}, \text{Customer B}) = 0.2$
- Grid Cell "Brand Loyalists":
 - o Customer C: $\mu(\text{"Brand Loyalists"}, \text{Customer C}) = 0.6$
 - o Customer D: $\mu(\text{"Brand Loyalists"}, \text{Customer D}) = 0.4$
- Grid Cell "Impulse Buyers":
 - o Customer E: $\mu(\text{"Impulse Buyers"}, \text{Customer E}) = 0.3$
 - o Customer F: $\mu(\text{"Impulse Buyers"}, \text{Customer F}) = 0.7$

Rule Generation:

Based on the fuzzy grid partitioning and the relevant attributes identified through rough set analysis, the following rules were generated:

- Rule 1: If a customer belongs to the grid cell "Value Shoppers" and their average purchase value is below \$50, then they are classified as "Value Shoppers."
- Rule 2: If a customer belongs to the grid cell "Brand Loyalists" and their purchase frequency of a specific brand is high, then they are classified as "Brand Loyalists."
- Rule 3: If a customer belongs to the grid cell "Impulse Buyers" and their satisfaction rating is above 4, then they are classified as "Impulse Buyers."

Discussion

The hybrid approach successfully segmented the customers into distinct segments based on their purchasing behavior. The fuzzy grid partitioning process divided the customer space into three grid cells, each representing a different segment. The fuzzy membership values assigned to the customers in each grid cell reflect the degree of association with that particular segment.

The generated rules provide actionable insights into the characteristics and behaviors of customers in each segment. For example, Rule 1 suggests that customers in the "Value Shoppers" segment have average purchase values below \$50. Rule 2 indicates that customers in the "Brand Loyalists" segment exhibit high purchase frequency of a specific brand. Rule 3 highlights that customers in the "Impulse Buyers" segment have high satisfaction ratings.

These rules can assist the retail company in tailoring marketing strategies, offering personalized promotions, and enhancing customer satisfaction. By understanding the specific preferences and behaviors of customers in each segment, the company can optimize its marketing efforts and improve customer targeting.

It's important to note that the effectiveness of the customer segmentation model should be evaluated through performance metrics, such as segment purity, accuracy, and business impact. Additionally, the model should be tested and validated on a larger dataset and compared with alternative segmentation approaches to assess its robustness and reliability.

The case example demonstrates the potential of the hybrid approach to effectively segment customers in a retail setting using adaptive fuzzy grid partitioning and rule generation. However, in real-world applications, additional factors, such as customer lifetime value, geographic location, and product preferences, can be considered to further enhance the segmentation model and its practical applicability.

Conclusion.

In this research, we proposed a hybrid approach for adaptive fuzzy grid partitioning and rule generation using rough set theory to address the problem of customer segmentation based on purchasing behavior in a retail company. The research aimed to minimize the fuzziness of the partitioning while maximizing the accuracy and interpretability of the generated rules. Through the numerical example and case study, we demonstrated the effectiveness of the hybrid approach in segmenting customers and generating actionable rules for marketing strategies. The fuzzy grid partitioning successfully divided the customer space into distinct segments, and the generated rules provided valuable insights into the characteristics and behaviors of customers in each segment. The research highlighted the importance of adaptive fuzzy grid partitioning in capturing the complex relationships and patterns in the dataset, as well as the utility of rough set theory in attribute reduction and rule induction. The hybrid approach allowed for the incorporation of both fuzzy partitioning and rough set analysis, resulting in a comprehensive and robust customer segmentation model. The results and discussions presented in this research emphasize the potential benefits of the

hybrid approach for retail companies, including targeted marketing campaigns, personalized recommendations, and improved customer satisfaction. By understanding the specific preferences and behaviors of different customer segments, companies can optimize their marketing efforts and enhance their overall business performance. It is important to note that the proposed hybrid approach is not limited to the retail industry and can be applied to various domains that require data-driven segmentation and rule generation. However, further research and validation are needed to assess the performance of the approach on larger and more diverse datasets and to compare it with alternative methods. This research contributes to the field of customer segmentation by proposing a hybrid approach that combines adaptive fuzzy grid partitioning and rule generation using rough set theory. The results demonstrate the potential for accurate and interpretable customer segmentation, laying the foundation for future research and applications in the field of data-driven marketing and customer analytics.

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