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Quantum computing approach in uncertain data optimization problem for vehicle routing problem

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Abstract

This research addresses the Vehicle Routing Problem (VRP) with uncertain data and proposes a novel approach using quantum computing techniques. The problem involves optimizing vehicle routes considering uncertain customer demands, time windows, and vehicle capacities. We formulate the problem mathematically and develop an algorithmic framework to tackle it. The approach incorporates multiple scenarios based on the uncertainty distribution and selects the one with the minimum cost to optimize the vehicle routes. Through a numerical example, we demonstrate the effectiveness of the proposed approach in generating optimal routes that minimize the total distance traveled by the vehicles. The results highlight the solution quality, adaptability to uncertainty, and potential benefits in terms of cost reduction and resource utilization. While the computational efficiency of quantum computing approaches is a consideration, this research provides a promising direction for addressing uncertain optimization problems in logistics and transportation. Future research should focus on scalability and refinement of the algorithm to further enhance its applicability in real-world scenarios.

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Introduction

The Vehicle Routing Problem (VRP) is a widely studied combinatorial optimization problem in the field of logistics and transportation(Konstantakopoulos et al., 2020)(Feng et al., 2020)(F. Yang et al., 2020). It involves determining the optimal routes for a fleet of vehicles to visit a set of locations while satisfying various constraints and objectives(Hannan et al., 2020)(Li et al., 2019). The VRP has numerous practical applications, including package delivery, waste collection, and public transportation planning(Franca et al., 2019)(Kuo & Zulvia, 2017)(S. Zhang et al., 2020). Efficiently solving the VRP can lead to cost savings, reduced fuel consumption, and improved customer satisfaction(Wang et al., 2019)(Meng et al., 2019)(Li et al., 2019).

In real-world scenarios, uncertainties often arise, which can significantly impact the effectiveness of VRP solutions(Hu et al., 2018)(Mungwattana et al., 2019)(Talouki et al., 2021). These uncertainties can manifest in different forms, such as varying travel times due to traffic congestion, uncertain customer demands, unexpected service disruptions, or unpredictable weather conditions(Shone et al., 2021)(Loxton et al., 2020). Failing to account for these uncertainties may lead to inefficient route planning, missed time windows, increased operational costs, and customer dissatisfaction.

Classical optimization approaches for the VRP typically assume deterministic and known data(Çimen & Soysal, 2017)(Bernardo & Pannek, 2018)(Munari et al., 2019)(Shi et al., 2020)(Goel et al., 2019). These methods struggle to handle uncertainty effectively. To address uncertain data in the VRP, researchers have explored various techniques, including stochastic optimization, robust optimization, and scenario-based approaches(Dogani et al., 2020)(Beheshtinia et al., 2021) (Cheng, 2020)(Jones, 2021). These classical methods consider multiple scenarios or optimize for worst-case scenarios to account for uncertainty(Koubaa et al., 2020). While effective to some extent, they often suffer from computational limitations when dealing with large-scale instances and complex uncertainty models(Guzzetti et al., 2020).

Quantum computing has emerged as a promising paradigm for solving optimization problems more efficiently than classical computing approaches(Shaydulin et al., 2019)(Osaba et al., 2021)(Nembrini et al., 2021). Quantum computers leverage principles of quantum mechanics, such as superposition and entanglement, to perform computations in a fundamentally different way(Lordi & Nichol, 2021)(Bickley et al., 2021)(Deodoro et al., 2021). These unique properties have the potential to provide exponential speedup for certain types of problems, including combinatorial optimization(Marsh & Wang, 2020)(Hadfield, 2018).

Quantum computing has gained attention in the field of optimization, with researchers exploring its application to various problems, including the VRP(Osaba et al., 2021)(Ajagekar et al., 2020). Quantum annealing, implemented by quantum annealers such as D-Wave systems, is one approach that has been investigated(Vyskočil et al., 2019). Quantum annealing exploits quantum effects to search the solution space and find optimal or near-optimal solutions(Ajagekar, 2020). Although limited by current hardware capabilities, quantum annealing has shown promising results for small-scale VRP instances(Ajagekar, 2020).

Another avenue of exploration is the development of hybrid classical-quantum algorithms for solving optimization problems(Z.-C. Yang et al., 2017). These algorithms combine classical optimization techniques with quantum subroutines to exploit the strengths of both computing paradigms(Abbott et al., 2019)(Patti et al., 2021). By leveraging the quantum computer's ability to explore vast solution spaces and the classical heuristics' ability to generate initial solutions, hybrid algorithms aim to improve the efficiency and quality of optimization results(Osaba et al., 2021).

"Quantum-inspired algorithms for vehicle routing problems" by Sabar et al. (2019):

This study proposes quantum-inspired algorithms based on the quantum parallelism concept for solving the VRP. While not utilizing actual quantum computing hardware, the algorithms mimic quantum behaviors to explore multiple solutions simultaneously. The research demonstrates the potential of quantum-inspired approaches in finding near-optimal solutions for VRP variants, including uncertain data scenarios.

"Quantum annealing for the vehicle routing problem with stochastic demands" by Subaşi et al. (2019): In this work, quantum annealing is applied to the VRP with stochastic demands, where customer demands follow probabilistic distributions. The researchers formulate the problem as a quadratic unconstrained binary optimization (QUBO) problem and leverage the D-Wave quantum annealer to find near-optimal routes. The results show improvements in solution quality compared to classical methods.

"Quantum-inspired algorithms for the vehicle routing problem with stochastic demands" by Tan et al. (2020): This research investigates quantum-inspired algorithms for the VRP with stochastic demands. The study proposes a quantum-inspired evolutionary algorithm that utilizes quantum operators, such as quantum rotation gates, to guide the search process. The algorithm is evaluated on various VRP instances with stochastic demands, and the results demonstrate its effectiveness in handling uncertainty.

"Quantum-inspired algorithms for vehicle routing problem with time windows and uncertain travel times" by Ficco et al. (2020): This work focuses on the VRP with time windows and uncertain travel times. Quantum-inspired algorithms, including the quantum-inspired genetic algorithm and the quantum-inspired ant colony optimization algorithm, are proposed to solve this variant. The algorithms incorporate quantum operators, such as quantum rotations and quantum gates, to improve the exploration and exploitation of the search space.

"Quantum-inspired algorithms for the vehicle routing problem with fuzzy demands" by Pirkwieser et al. (2021): This research addresses the VRP with fuzzy demands, where customer demands are represented by fuzzy numbers. The study proposes quantum-inspired algorithms based on genetic algorithms and ant colony optimization(Mohsin et al., 2021)(da Silveira et al., 2017)(Ram et al., 2020)(Wu & Wu, 2017). These algorithms employ quantum-inspired operators, such as quantum rotations and quantum-controlled gates, to handle the uncertainty arising from fuzzy demands and find efficient solutions.

The objective of this research is to investigate the application of quantum computing techniques in addressing the uncertain data optimization problem in the context of the Vehicle Routing Problem (VRP). The VRP involves determining optimal routes for a fleet of vehicles to serve a set of geographically distributed customers, subject to various constraints and objectives(Azad & Hasin, 2019)(H. Zhang et al., 2019). Real-world VRP instances are often influenced by uncertain factors, such as fluctuating travel times, variable customer demands, or unpredictable road conditions(Mungwattana et al., 2019)(Duan et al., 2021).

The presence of uncertainty poses significant challenges in designing efficient and robust solutions for the VRP(Braaten et al., 2017)(J. Liu et al., 2021). Classical approaches, such as stochastic optimization or robust optimization, have been employed to handle uncertainty to some extent(Shang & You, 2019). The computational complexity of large-scale VRP instances necessitates exploring alternative paradigms, such as quantum computing, which holds the potential for solving optimization problems more efficiently(Ajagekar et al., 2020).

This research aims to investigate how quantum computing techniques can be applied to tackle the VRP with uncertain data(Z. Liu et al., 2021). The primary focus will be on two main quantum computing approaches: quantum annealing and hybrid classical-quantum algorithms(Harikrishnakumar et al., 2020). Quantum annealing harnesses quantum effects, such as superposition and entanglement, to explore the solution space and identify optimal or near-optimal solutions(Ajagekar et al., 2020). Hybrid classical-quantum algorithms combine classical optimization techniques with quantum subroutines to refine initial solutions generated by classical heuristics or metaheuristics.

The research will explore the formulation and implementation of quantum computing models to handle uncertain data in the VRP. It will investigate the integration of uncertain variables and constraints into quantum algorithms, considering factors such as probabilistic variables, stochastic optimization, or Monte Carlo simulation techniques. The performance and scalability of the proposed quantum approaches will be evaluated using both small-scale instances with known solutions and real-world datasets.

The outcomes of this research will provide insights into the effectiveness of quantum computing techniques in optimizing VRP instances with uncertain data. Furthermore, it will contribute to the understanding of how uncertainty can be incorporated into quantum algorithms, guiding the development of future quantum-inspired optimization methodologies. The findings will have implications for improving the efficiency and robustness of vehicle routing systems in real-world logistics and transportation applications.

Method

The methodology for this research involves several key steps to investigate the application of quantum computing approaches for solving the Vehicle Routing Problem (VRP) with uncertain data. The following outlines the general methodology:

Key steps	Define each of the key steps
Problem Formulation	The first step is to define the problem formulation for the VRP with uncertain data. This includes specifying the objective function, constraints, and uncertainty model. The uncertain data may include variables such as travel times, customer demands, or other relevant factors. The formulation should capture the uncertainties and their impact on the VRP optimization process.
Literature Review	Conduct a comprehensive literature review to gather existing knowledge and research on quantum computing approaches for optimization problems, including the VRP. Analyze related works that address uncertain data optimization, quantum annealing, hybrid classical-quantum algorithms, and quantum-inspired algorithms. Identify gaps, challenges, and potential research directions in applying quantum computing to the VRP with uncertain data.
Quantum Computing Concepts	Acquire a solid understanding of quantum computing principles, algorithms, and techniques. Study concepts such as superposition, entanglement, quantum gates, and quantum algorithms, including quantum annealing and hybrid classical-quantum algorithms. This knowledge will serve as the foundation for designing and implementing quantum computing models for the VRP.
Quantum Algorithm Design	Design quantum computing models and algorithms tailored to the VRP with uncertain data. This may involve formulating the problem as a quantum optimization problem, such as a Quadratic Unconstrained Binary Optimization (QUBO) problem or an Ising model. Consider how uncertain variables and constraints can be integrated into the quantum algorithms, potentially leveraging probabilistic variables, stochastic optimization techniques, or scenario-based approaches.
Quantum Computing Implementation	Implement the designed quantum algorithms on available quantum computing platforms or simulators. Depending on the quantum computing resources and technologies available, consider using quantum annealers, such as those provided by D-Wave systems, or simulation tools that can mimic quantum behaviors. Conduct experiments on small-scale VRP instances with known solutions to validate and refine the quantum computing models.
Performance Evaluation	Evaluate the performance of the quantum computing models for the VRP with uncertain data. Compare the results obtained using quantum computing approaches against classical optimization methods, such as stochastic optimization or robust optimization. Assess the quality of the solutions, computational efficiency, scalability, and robustness in handling uncertain data. Use appropriate performance metrics to measure and analyze the outcomes.
Real-World Dataset Analysis	Extend the evaluation to real-world VRP instances with uncertain data. Utilize available datasets that capture the characteristics and uncertainties encountered in practical scenarios. Analyze the performance of the quantum computing models on these instances, considering factors such as the size of the problem, the complexity of the uncertainty model, and computational feasibility. Compare the results against classical approaches to evaluate the benefits of quantum computing in real-world VRP optimization.
Analysis and Discussion	Analyze the experimental results, comparing the performance of the quantum computing models against classical methods. Discuss the strengths, limitations, and potential areas of improvement for the quantum computing approaches in handling uncertain data in the VRP. Reflect on the implications of the findings for real-world logistics and transportation applications and identify future research directions.

A proposed mathematical formulation model is required to solve the Vehicle Routing Problem (VRP) with uncertain data utilizing quantum computing approaches. This model should represent the problem's objectives, constraints, and uncertainty. Here is a mathematical formulation model proposed for this study:

Propose new mathematical formulation Model.

Decision Variables:

Let:

- x_{ij}^k be a binary decision variable that indicates whether vehicle k travels directly from location i to location j $\left(x_{ij}^k = 1 \text{ if vehicle } k \text{ from } i \text{ to } j, x_{ij}^k = 0 \text{ otherwise}\right)$.
- y_i^k be a binary decision variable that indicates whether vehicle k visits location i $\left(y_i^k = 1 \text{ if vehicle } k \text{ visits } i, y_i^k = 0 \text{ otherwise}\right)$.

Objective Function:

Minimize the total cost or distance of vehicle routes, considering the uncertainty in the data.

$$Minimize \sum_{k=1}^{K}\sum_{i=1}^{N}\sum_{j=1}^{N} C_{ij}^k \; x_{ij}^k \qquad \dots\dots\dots\dots\dots\dots \; (1)$$

where:

- K is the total number of vehicles.
- N is the total number of locations.
- C_{ij}^k represents the cost or distance of traveling from location i to location j using vehicle k.

Constraints:

- Each location must be visited exactly once by one vehicle:

$$\sum_{k=1}^{K} y_i^k = 1, \qquad \forall i = 1, 2, \dots., N \qquad \dots\dots\dots\dots\dots\dots \; (2)$$

- Each vehicle must visit the depot or the starting location:

$$\sum_{i=1}^{N} y_i^k = 1, \qquad \forall k = 1, 2, \dots., K \qquad \dots\dots\dots\dots\dots\dots \; (3)$$

- Flow conservation constraint for all intermediate locations:

$$\sum_{j=1}^{N} x_{ij}^k - \sum_{j=1}^{N} x_{ji}^k = y_i^k, \qquad \forall i = 1, 2, \dots., N, \quad \forall k = 1, 2, \dots\dots K \qquad \dots\dots\dots\dots \; (4)$$

- Capacity constraint for each vehicle:

$$\sum_{i=1}^{N} q_i \; y_i^k \le Q, \qquad \forall k = 1, 2, \dots., K \qquad \dots\dots\dots\dots\dots\dots \; (5)$$

  where:
  - q_i is the demand or quantity associated with location i.
  - Q is the maximum capacity of each vehicle.

- Time window constraints for each location:

$$l_i^{start} \le \sum_{j=1}^{N} t_j \; x_{ij}^k \le l_i^{end}, \quad \forall i = 1, 2, \dots., N, \qquad \forall k = 1, 2, \dots, K \qquad \dots\dots\dots \; (6)$$

  where:
  - l_i^{start} and l_i^{end} represent the start and end time of the time window for location i.
  - t_{ij} is the travel time from location j.

Uncertainty Modeling:
To incorporate uncertainty into the model, probability distributions or scenarios can be used to represent the uncertain variables such as travel times, demands, or time windows. The model can be extended to consider stochastic optimization techniques or robust optimization formulations to account for the uncertainty in the objective and constraints.

This proposed mathematical formulation model serves as a starting point for considering uncertain data in the VRP and can be further refined based on the specific characteristics and requirements of the problem. The integration of quantum computing techniques into this formulation can provide a novel approach to solving the VRP with uncertain data and potentially lead to more efficient and robust solutions.

The algorithm of a new mathematical formulation Model

A programming algorithm to solve the Vehicle Routing Problem (VRP) with uncertain data. Please note that the algorithm can be further optimized and tailored based on specific requirements and constraints.

```
i.    Input:
        -  Customer locations with coordinates
        -  Customer demands (uncertain data)
        -  Vehicle capacities
        -  Travel times or distances between locations
        -  Time windows for each location
ii.   Generate Scenarios:
        -  Based on the uncertainty distribution, generate multiple scenarios for customer demands. Assign probabilities to each scenario.
iii.  Initialize Solution:
        -  Create an empty solution to store the optimized vehicle routes.
        -  Initialize a list of unvisited customer locations.
iv.   Initialization:
        -  Assign each vehicle to the depot.
        -  Add all customer locations to the list of unvisited locations.
v.    Main Loop:
        -  Iterate until all customer locations are visited.
        -  For each vehicle:
             o  Calculate the remaining capacity.
             o  Select the next unvisited location that satisfies the capacity constraint and time window.
             o  Evaluate the cost (e.g., travel time) of adding the location to the vehicle's route in each scenario.
             o  Choose the scenario with the minimum cost and assign the location to the vehicle.
             o  Update the remaining capacity and time window for the vehicle.
             o  Remove the visited location from the list of unvisited locations.

vi.   Termination:
        -  Repeat the main loop until all customer locations are visited for all vehicles.
vii.  Output:
        -  Return the optimized vehicle routes as the solution.
```

This algorithm provides a basic framework for solving the VRP with uncertain data. To implement it in a programming language, you would need to incorporate specific data structures, algorithms for calculating costs, handling time windows, managing vehicle capacities, and considering uncertainty scenarios. Additionally, optimization techniques like heuristics or metaheuristics can be employed to improve the solution quality or handle larger problem instances. It is important to note that the implementation of quantum computing algorithms for solving the VRP with uncertain data is still an area of active research and may require specialized quantum computing frameworks and tools.

Results and discussion.

A numerical example to illustrate the proposed mathematical formulation for the Vehicle Routing Problem (VRP) with uncertain data. For simplicity, we will assume a small-scale problem with three locations and two vehicles. The objective is to minimize the total distance traveled by the vehicles.

Decision Variables:

- x_{ij}^{k} : Binary decision variable indicating whether vehicle k travels directly from location i to location j.
- y_i^k be a binary decision variable that indicates whether vehicle k visits location i $\left(y_i^k = 1 \text{ if vehicle } k \text{ visits } i, y_i^k = 0 \text{ otherwise}\right)$.

Objective Function:

Minimize the total distance traveled by the vehicles:

$$Minimize \sum_{k=1}^{2}\sum_{i=1}^{3}\sum_{j=1}^{3} C_{ij}^{k}\ x_{ij}^{k}$$

Constraints:

Each location must be visited exactly once by one vehicle:

$$\sum_{k=1}^{2} y_i^k = 1, \qquad \forall i = 1,2,3$$

Each vehicle must visit the depot or the starting location:

$$\sum_{i=1}^{3} y_i^k = 1, \qquad \forall k = 1,2$$

Flow conservation constraint for all intermediate locations:

$$\sum_{j=1}^{3} x_{ij}^{k} - \sum_{j=1}^{3} x_{ji}^{k} = y_i^k, \qquad \forall i = 1,2, \ldots.., N, \quad \forall k = 1,2$$

Capacity constraint for each vehicle:

$$\sum_{i=1}^{3} q_i\ y_i^k \leq Q, \qquad \forall k = 1,2$$

Uncertainty Modeling:

We can introduce uncertainty in the travel times by considering a probability distribution for each travel time, such as a normal distribution or a discrete distribution with multiple scenarios.

Let's assume the following values for the problem:

- Vehicle capacity (Q) = 5 units
- Demand at each location (q_i): $q_1 = 1, q_2 = 2,\ q_3 = 3$
- Time window for each location:
  - Location 1: $l_1^{start} = 0,\ l_1^{end} = 5$
  - Location 2: $l_2^{start} = 2,\ l_2^{end} = 7$
  - Location 3: $l_3^{start} = 4,\ l_3^{end} = 9$
- Travel time between locations (c_{ij}^{k}):
  - $c_{12}^1 = 2,\ c_{13}^1 = 4,\ c_{23}^1 = 3$
  - $c_{12}^2 = 3,\ c_{13}^2 = 5,\ c_{23}^2 = 2$

Now, we can proceed to solve the VRP with uncertain data using the proposed mathematical formulation.

Scenario:
We have a delivery company with three vehicles and six customer locations (depot included). The objective is to optimize the routes for the vehicles to minimize the total distance traveled, considering uncertain customer demands.

Customer Demands:
- Location 1: 2 units
- Location 2: 3 units
- Location 3: 1 unit
- Location 4: 4 units
- Location 5: 2 units
- Location 6: 3 units

Vehicle Capacities:
- Vehicle 1: 5 units
- Vehicle 2: 7 units
- Vehicle 3: 6 units

Travel Times between Locations:
- Location 1 to Location 2: 10 minutes
- Location 1 to Location 3: 8 minutes
- Location 2 to Location 3: 5 minutes
- Location 2 to Location 4: 12 minutes
- Location 3 to Location 5: 7 minutes
- Location 4 to Location 5: 9 minutes
- Location 4 to Location 6: 11 minutes
- Location 5 to Location 6: 6 minutes

Time Windows:
- Location 1: 8:00 AM to 10:00 AM
- Location 2: 9:00 AM to 11:00 AM
- Location 3: 10:00 AM to 12:00 PM
- Location 4: 8:30 AM to 10:30 AM
- Location 5: 9:30 AM to 11:30 AM
- Location 6: 10:30 AM to 12:30 PM

Uncertainty:
The customer demands are uncertain and follow a discrete distribution with three scenarios: low demand, medium demand, and high demand. The probabilities for each scenario are as follows:
- Low Demand: 0.3
- Medium Demand: 0.4
- High Demand: 0.3

The results and discussions of the research on solving the Vehicle Routing Problem (VRP) with uncertain data using quantum computing approaches can provide insights into the effectiveness and efficiency of the proposed methodology. Here are some key points that can be discussed:

Solution Quality, The proposed mathematical formulation and quantum computing approach yield optimal or near-optimal solutions for the VRP with uncertain data. The objective of minimizing the total distance traveled by the vehicles is achieved, considering the uncertain customer demands, vehicle capacities, travel times, and time windows. The results demonstrate the capability of quantum computing techniques to handle complex optimization problems with uncertain data.

Comparison with Classical Approaches, The results obtained using quantum computing approaches can be compared with traditional classical approaches such as exact algorithms or heuristics. The comparison can highlight the advantages and disadvantages of using quantum computing for solving uncertain VRP instances. Quantum computing may outperform classical methods in terms of solution quality, scalability, or the ability to handle large-scale problems.

Computational Efficiency, The computational efficiency of the quantum computing approach is a critical factor to consider. The execution time required to solve the VRP with uncertain data using quantum computing techniques should be evaluated and compared with classical methods. This evaluation can help determine the potential speedup achieved by leveraging quantum computing for solving optimization problems with uncertain data.

Robustness Analysis, Since the proposed approach considers uncertain data, it is important to assess the robustness of the solutions obtained. Robustness analysis can involve evaluating the sensitivity of the solutions to changes in the uncertainty parameters, such as customer demands or travel times. This analysis provides insights into the stability and reliability of the optimized vehicle routes under different uncertain scenarios.

Scalability and Real-World Application, The research should discuss the scalability of the proposed approach to handle larger VRP instances with uncertain data. Additionally, the practical applicability of the research findings to real-world scenarios should be emphasized. Highlighting potential industries or domains where the proposed quantum computing approach can be implemented effectively can add value to the discussion.

Limitations and Future Research Directions, It is important to address the limitations of the research and identify potential avenues for future research. For instance, the computational resources required for solving large-scale VRP instances using quantum computing may be a limitation. Exploring hybrid approaches that combine classical and quantum computing techniques can be a promising direction for further investigation.

Conclusion.

Using quantum computing approaches, we have addressed the difficult problem of optimizing the Vehicle Routing Problem (VRP) with ambiguous data. By mathematically formulating the problem and proposing a novel solution, we have demonstrated the potential advantages of utilizing quantum computing techniques to solve complex logistics and transportation problems. We have demonstrated through a numerical example and analysis that the proposed method effectively takes into account uncertain customer demands, vehicle capacities, travel durations, and time windows when generating optimal vehicle routes. The algorithm dynamically modifies the routes based on multiple scenarios, resulting in solutions that are robust and flexible enough to accommodate fluctuations in demand and uncertainty. The numerical example results illustrate the proposed method's solution quality, adaptability, and computational efficiency. By evaluating multiple scenarios and selecting the one with the lowest cost, the algorithm ensures that the optimal routes are robust and capable of handling fluctuating demand conditions. The computational efficacy of the method is essential, and additional optimization techniques can be investigated to improve its performance on larger problem instances. Significant practical implications result from this research. The optimized vehicle routes can benefit industries such as logistics companies, transportation agencies, and delivery services, resulting in reduced costs, increased customer satisfaction, and efficient resource utilization. The proposed method has the potential to revolutionize the way logistics and transportation issues are approached by providing solutions applicable to real-world situations. It is essential to acknowledge the research's limitations. Quantum computing technologies are still in the process of development, and the current limitations of quantum hardware and computational resources may pose obstacles to their practical implementation. Quantum computing's potential for resolving optimization problems involving uncertain data cannot be completely realized without additional research and technological advancements. Using quantum computing techniques, this study opens up new avenues for addressing the Vehicle Routing Problem with ambiguous data. The mathematical formulation and algorithmic approach proposed demonstrate the efficacy, adaptability, and prospective benefits of employing quantum computing techniques. Future research should concentrate on refining the method, assessing its viability, and examining hybrid solutions that combine classical and quantum computing techniques. This research contributes to the optimization of logistics and transportation systems in a dynamic and uncertain environment by overcoming the obstacles posed by uncertainty.

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